Dynamics of Cable Structures

K. Yu. Volokh[1]; O. Vilnay[2]; and I. Averbuh[3]

Abstract: Initial infinitesimal modes of rigid body motions are used to form a reduced basis for nonlinear dynamic analysis of cable structures. This approach superimposed on the geometrically nonlinear truss formulation extracts slow motions from the general dynamic response of cable systems. In this way the problem is reduced considerably and solution of the equations becomes smoother. These two features are computationally desirable. The advantage of the proposed procedure is studied using numerical examples of a plane cable net and a cut-down version of the Geiger dome. Problems of time-history computation and periodic motion analysis are addressed in the examples.

DOI: 10.1061/(ASCE)0733-9399(2003)129:2(175)

CE Database keywords: Cables; Dynamics; Analysis.

Introduction

A linear approach to dynamic analysis of cable structures is based on linearized dynamic equations, where the initial tangent stiffness matrix, comprising the uncoupled linear elastic stiffness matrix and the geometric stiffness matrix, is used (Krishna 1978; Buchholdt 1985). This is in contrast to the well-known fact that the static response of cable structures is generally nonlinear (Irvine 1981; Szabo and Kollar 1984). The necessity to treat general nonlinear analysis of dynamics of cable structures has been emphasized by Mesarovic and Gasparini (1992a,b) although some interesting work was also carried out by Fried (1982); Milinazzo et al. (1987); Perkins and Mote (1987); Triantafyllou and Howell (1992); Kahla (1995); Koh et al. (1999); and Hu and Jin (2001).

Mesarovic and Gasparini (1992a,b) raised questions about the validity of linear analysis that considers dynamic loads and the nonlinear dynamic response of cable structures. Their first paper (Mesarovic and Gasparini 1992a) is devoted to a search of an adequate nonlinear model for an eight-element cable system. Using geometric reasoning specific to this cable system they extracted the dominant displacement modes (generalized coordinates). These modes are further used to simplify dynamic equations. In the second paper (Mesarovic and Gasparini 1992b) nonlinear dynamic response of the same system including a parametric study and the nonlinear stability issue is analyzed. The conclusion from these studies is that linear dynamic analysis is not adequate, "most of the nonlinear dynamic phenomena observed cannot be predicted by adding the geometric stiffness of the elements and analyzing the resultant equations of motion."

In the present work a general approach is developed for the nonlinear dynamic modeling of arbitrary cable structures. This approach is based on the reduced basis technique. The idea behind reduced basis techniques may be described briefly as follows: let the physical phenomenon be represented by the nonlinear operator equation:

$$\boldsymbol{R}[\boldsymbol{u}]=0 \tag{1}$$

where $\boldsymbol{u}$ is an unknown. Then the unknown is approximated by the finite series:

$$\boldsymbol{u}=\sum_{i=1}^{n} a_i \boldsymbol{u}_i \tag{2}$$

where a_i=coefficient to be found; and $\boldsymbol{u}_i$=assumed mode. Substituting Eq. (2) into Eq. (1) and pre-multiplying the latter scalarly by $\boldsymbol{u}_i$, it is possible to obtain a new set of n nonlinear algebraic equations with n unknowns a_i:

$$\left\langle \boldsymbol{u}_i, \boldsymbol{R}\left[\sum_{j=1}^{n} a_j \boldsymbol{u}_j\right]\right\rangle=0, \quad i=1,\ldots,n \tag{3}$$

This set of equations is solved by existing numerical methods. If $\boldsymbol{u}$ belongs to an appropriate function space then the technique described is a general discretization scheme and various well-known methods (finite elements, differences and etc.) may be interpreted in this way. This may also be considered the first stage in reducing the size of the problem. Further reduction can be achieved by using a reduced basis for the already discrete system of algebraic equations, Eq. (1), where $\boldsymbol{u}$ belongs to m-dimensional real Euclidean space in which $n<m$. In this case Eq. (3) presents further reduction of the initial problem. This reduction is, in the narrow sense, a *reduced basis technique*. A large amount of work was carried out on various methods to establish appropriate reduced bases. A state-of-the-art review was given by Noor (1994). General methods of establishing a reduced basis are often unreliable and the choice of reduced basis must proceed on a problem-by-problem basis.

In this paper use of initial infinitesimal modes of rigid body motion as the reduced basis for modeling nonlinear dynamics of cable structures is proposed. This reduced basis approach is superimposed on the nonlinear truss formulation (Szabo and Kollar 1984; Argyris and Mlejnek 1991; Crisfield 1991; Volokh 2001).

[1]Senior Lecturer, Dept. of Civil Engineering, Technion, Haifa 32000, Israel. E-mail: cvolokh@aluf.technion.ac.il

[2]Associate Professor, Dept. of Civil Engineering, Technion, Haifa 32000, Israel.

[3]Graduate Student, Dept. of Civil Engineering, Technion, Haifa 32000, Israel.

Note. Associate Editor: James L. Beck. Discussion open until July 1, 2003. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on March 17, 2000; approved on June 11, 2002. This paper is part of the ***Journal of Engineering Mechanics***, Vol. 129, No. 2, February 1, 2003. ©ASCE, ISSN 0733-9399/2003/2-175–180/$18.00.

Physically, the use of the proposed reduced basis allows averaging the dynamical response of cable systems and extracting the most important slow motions. Numerically, this means a significant reduction of the size of the problem and smoothing of the solution of dynamic equations. These numerical features are computationally desirable. Two numerical examples are considered. The first one is a prestressed two-dimensional (2-D) cable net comprising 11 members (Fig. 2). The second one is a prestressed three-dimensional (3-D) dome comprising 8 vertical struts and 28 cables (Fig. 3). This dome is a cut-down version of the Geiger dome (Geiger 5 year): the Gymnastic Arena and the Fencing Arena built in Korea, and the Redbird Arena at Illinois State Univ. and the Sun Coast Dome in St. Petersburg, Fla.

Governing Equations

General Equations of Nonlinear Dynamics

Dynamic equations without damping take the form

$$\boldsymbol{R}[\boldsymbol{u}]=\boldsymbol{M}\ddot{\boldsymbol{u}}+\boldsymbol{r}[\boldsymbol{u}]-\boldsymbol{q}=0 \tag{4}$$

where $\boldsymbol{M}=m\times m$ diagonal lumped mass matrix, or a consistent nondiagonal matrix when the finite element approximation is used; $\boldsymbol{u}=[t]$ and $\boldsymbol{q}=\boldsymbol{q}[t]$ are m dimensional vectors of nodal displacements and external nodal forces; and vector $\boldsymbol{r}$ of the internal nodal forces may be computed by the strain energy expression (Volokh 1999):

$$\boldsymbol{r}[\boldsymbol{u}]=\frac{\partial\Omega[\boldsymbol{u}]}{\partial\boldsymbol{u}},\quad r_j=\frac{\partial\Omega}{\partial u_j} \tag{5}$$

$$\Omega[\boldsymbol{u}]=\tfrac{1}{2}\Delta^T[\boldsymbol{u}]\boldsymbol{S}\Delta[\boldsymbol{u}]+\boldsymbol{p}_0^T\Delta[\boldsymbol{u}] \tag{6}$$

$\boldsymbol{p}_0=k$ dimensional vector of initial member forces (prestressing); Δ=vector of member elongations; and $\boldsymbol{S}=k\times k$ uncoupled stiffness matrix with diagonal nonzero entries: $S_{ii}=E_iA_i/l_i$ including the i-th member Young's modulus, cross-sectional area and length, respectively.

Taking into account Hooke's law of member force increments $\boldsymbol{p}=\boldsymbol{S}\Delta$, Eq. (5) takes the form

$$\boldsymbol{r}[\boldsymbol{u}]=\frac{\partial\Omega[\boldsymbol{u}]}{\partial\boldsymbol{u}}=\boldsymbol{B}^T[\boldsymbol{u}](\boldsymbol{p}_0+\boldsymbol{p}[\boldsymbol{u}]),\quad \boldsymbol{B}[\boldsymbol{u}]=\frac{\partial\Delta[\boldsymbol{u}]}{\partial\boldsymbol{u}} \tag{7}$$

where $\boldsymbol{B}=k\times m$ kinematic matrix, and its transpose is the equilibrium matrix.

The "small strain" approximation implies

$$\Delta_i=l_i\varepsilon_i,\quad \varepsilon_i=e_i+\omega_i$$

$$e_i=\frac{x_j-x_s}{l_i}\frac{u_j-u_s}{l_i}+\frac{x_{j+1}-x_{s+1}}{l_i}\frac{u_{j+1}-u_{s+1}}{l_i}+\frac{x_{j+2}-x_{s+2}}{l_i}\frac{u_{j+2}-u_{s+2}}{l_i}$$

$$\omega_i=\frac{1}{2}\left(\frac{u_j-u_s}{l_i}\right)^2+\frac{1}{2}\left(\frac{u_{j+1}-u_{s+1}}{l_i}\right)^2+\frac{1}{2}\left(\frac{u_{j+2}-u_{s+2}}{l_i}\right)^2$$

where u_j and x_j=appropriate nodal displacements and coordinates of the i-th member.

Linear dynamic equations are obtained by linearizing $\boldsymbol{r}$ with respect to $\boldsymbol{u}$ at the initial configuration $\boldsymbol{u}=0$:

$$\boldsymbol{r}[\boldsymbol{u}]=\boldsymbol{K}\boldsymbol{u} \tag{8}$$

where $\boldsymbol{K}$=initial tangent stiffness matrix or, simply, the stiffness matrix:

$$\boldsymbol{K}=\left.\frac{\partial^2\Omega}{\partial\boldsymbol{u}\partial\boldsymbol{u}}\right|_{\boldsymbol{u}=0}=\boldsymbol{A}+\boldsymbol{D},\quad K_{js}=\left.\frac{\partial^2\Omega}{\partial u_s\partial u_j}\right|_{u_j=0}=A_{js}+D_{js} \tag{9}$$

$\boldsymbol{A}$=uncoupled linear elastic stiffness matrix; and $\boldsymbol{D}$=geometric stiffness matrix depending on the prestressing forces:

$$\boldsymbol{A}=\boldsymbol{B}^T\boldsymbol{S}\boldsymbol{B}|_{\boldsymbol{u}=0},\quad A_{js}=\sum_{l=1}^{k}\left.\frac{\partial\Delta_l}{\partial u_j}\frac{\partial\Delta_l}{\partial u_s}S_{ll}\right|_{u_j=0} \tag{10}$$

$$\boldsymbol{D}=\left.\frac{\partial[\boldsymbol{B}^T\boldsymbol{p}_0]}{\partial\boldsymbol{u}}\right|_{\boldsymbol{u}=0},\quad D_{js}=\sum_{l=1}^{k}\left.\frac{\partial^2\Delta_l}{\partial u_j\partial u_s}p_{0l}\right|_{u_j=0} \tag{11}$$

The proper index notation is used above in order to clarify matrix abbreviations.

It is worth emphasizing that the formulation of dynamic equations given above for cable structures is based on the direct use of global nodal degrees of freedom and the strain energy expression. It is believed that this method is the clearest and shortest. However, the same equations may be obtained using standard finite element procedures for geometrically nonlinear truss formulation (Szabo and Kollar 1984; Argyris and Mlejnek 1991; Crisfield 1991). When these methods are used, tedious preparations, including local element formulation, transition to global formulation and assembling of elements are required.

Time History: Initial Value Problem

In order to trace the time-history response of a specific structure it is necessary to add initial conditions to the dynamic equations:

$$\boldsymbol{u}|_{t=0}=\boldsymbol{a},\quad \dot{\boldsymbol{u}}|_{t=0}=\boldsymbol{b} \tag{12}$$

By introducing new unknowns $\boldsymbol{v}=\{\boldsymbol{u}^T,\dot{\boldsymbol{u}}^T\}^T$, which include nodal velocities, it is possible to obtain the canonical form of the initial value problem:

$$\boldsymbol{G}[\boldsymbol{v}]=\dot{\boldsymbol{v}}-\boldsymbol{f}[\boldsymbol{v}]=0 \tag{13}$$

$$\boldsymbol{v}|_{t=0}=\boldsymbol{c} \tag{14}$$

where $\boldsymbol{c}=\{\boldsymbol{a}^T,\boldsymbol{b}^T\}^T$.

Periodic Motion: Two-Point Boundary Value Problem

It is possible to search for periodic solutions of Eq. (4) or (13). In this case dynamic equations should be supplemented with periodic conditions:

$$\boldsymbol{u}[t]=\boldsymbol{u}[t+T],\quad \dot{\boldsymbol{u}}[t]=\dot{\boldsymbol{u}}[t+T] \tag{15}$$

or

$$\boldsymbol{v}[t]=\boldsymbol{v}[t+T] \tag{16}$$

In this case Eqs. (4) and (13) with conditions (15) and (16) take the form of the two-point boundary value problem (Seydel 1994; Nayfeh and Barachandran 1995).

Displacement Modes and Reduced Basis

The main feature of cable structures is the existence of infinitesimal modes of rigid body motion. Mathematically, this means that the linearized kinematic equations possess nontrivial solutions:

$$\boldsymbol{B}_0\boldsymbol{x}=0,\quad \boldsymbol{B}_0\equiv\boldsymbol{B}|_{\boldsymbol{u}=0} \tag{17}$$

$$\boldsymbol{x}=\boldsymbol{u}_1z_1+\boldsymbol{u}_2z_2+\cdots+\boldsymbol{u}_nz_n=\boldsymbol{U}\boldsymbol{z} \tag{18}$$

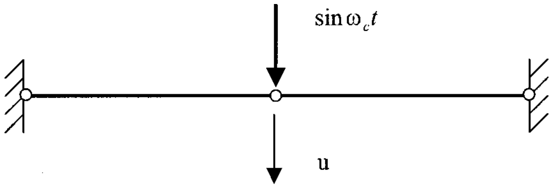

Fig. 1. One degree of freedom assembly with resonance loading

where vectors $\boldsymbol{u}_i$ form an orthonormal basis of the nullspace of matrix $\boldsymbol{B}_0$. By completing this subspace with its orthogonal complement

$$\hat{\boldsymbol{U}} = \{\boldsymbol{u}_{n+1}|\cdots|\boldsymbol{u}_m\}, \quad \boldsymbol{U}^T\hat{\boldsymbol{U}} = 0 \tag{19}$$

it is possible to represent the vector of nodal displacements in the following form:

$$\boldsymbol{u} = \boldsymbol{U}\boldsymbol{z} + \hat{\boldsymbol{U}}\hat{\boldsymbol{z}} \tag{20}$$

where $\boldsymbol{z}$ and $\hat{\boldsymbol{z}}$=new unknown vectors.

The first term on the right-hand side of Eq. (20) presents the infinitesimal rigid body motion. These modes affect the whole structural response (Volokh and Vilnay 1997). To show this let the following linearized equilibrium equation be considered:

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{q} \tag{21}$$

Using Eq. (20) and premultiplying by matrix $\{\hat{\boldsymbol{U}}|\boldsymbol{U}\}^T$ from the left, Eq. (21) takes the form

$$\begin{Bmatrix} \boldsymbol{K}_1 & \boldsymbol{L} \\ \boldsymbol{L}^T & \boldsymbol{K}_2 \end{Bmatrix} \begin{Bmatrix} \hat{\boldsymbol{z}} \\ \boldsymbol{z} \end{Bmatrix} = \begin{Bmatrix} \hat{\boldsymbol{U}}\boldsymbol{q} \\ \boldsymbol{U}\boldsymbol{q} \end{Bmatrix} \tag{22}$$

or

$$\begin{aligned} (\boldsymbol{K}_1 - \boldsymbol{L}\boldsymbol{K}_2^{-1}\boldsymbol{L}^T)\hat{\boldsymbol{z}} &= (\hat{\boldsymbol{U}}^T - \boldsymbol{L}\boldsymbol{K}_2^{-1}\boldsymbol{U}^T)\boldsymbol{q} \\ (\boldsymbol{K}_2 - \boldsymbol{L}^T\boldsymbol{K}_1^{-1}\boldsymbol{L})\boldsymbol{z} &= (\boldsymbol{U}^T - \boldsymbol{L}^T\boldsymbol{K}_1^{-1}\hat{\boldsymbol{U}}^T)\boldsymbol{q} \end{aligned} \tag{23}$$

where

$$\boldsymbol{K}_1 = \hat{\boldsymbol{U}}^T\boldsymbol{K}\hat{\boldsymbol{U}} = \hat{\boldsymbol{U}}^T\boldsymbol{B}_0^T\boldsymbol{S}\boldsymbol{B}_0\hat{\boldsymbol{U}} + \hat{\boldsymbol{U}}^T\boldsymbol{D}\hat{\boldsymbol{U}}$$

$$\boldsymbol{K}_2 = \boldsymbol{U}^T\boldsymbol{K}\boldsymbol{U} = \boldsymbol{U}^T\boldsymbol{D}\boldsymbol{U}$$

$$\boldsymbol{L} = \hat{\boldsymbol{U}}^T\boldsymbol{K}\boldsymbol{U} = \hat{\boldsymbol{U}}^T\boldsymbol{D}\boldsymbol{U}$$

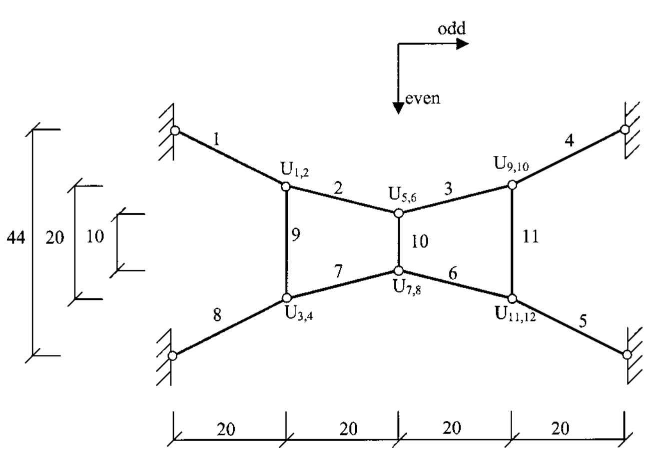

Fig. 2. Plane cable net

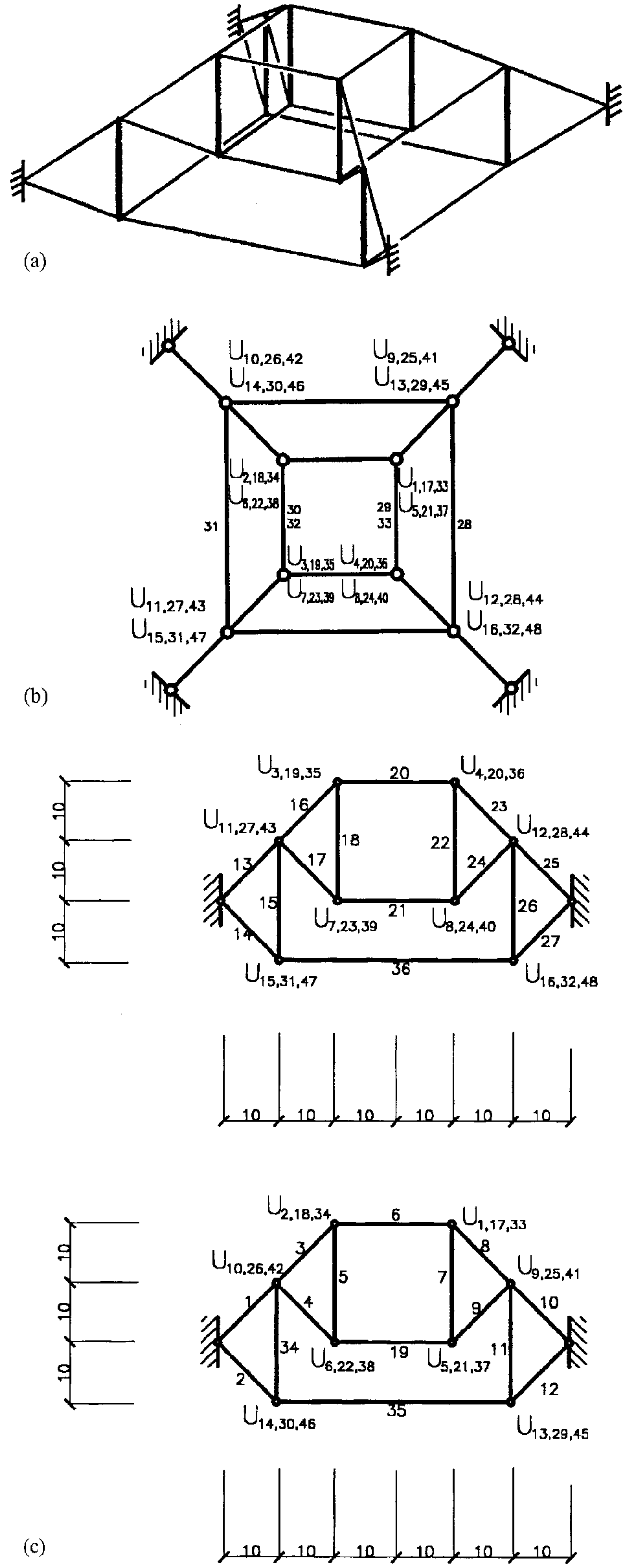

Fig. 3. Perspective (a), top (b), and front (c) views of a cable dome. Subscripts designate nodal degrees of freedom in three perpendicular directions. All vertical members are struts. Other members are cables.

Taking into account that the initial member stresses are significantly smaller than the elasticity modulus $\|\boldsymbol{D}\| \ll \|\boldsymbol{S}\|$ and consequently $\|\boldsymbol{K}_1\| \gg \|\boldsymbol{K}_2\| \sim \|\boldsymbol{L}\|$, it is possible to conclude from Eq. (23) that $\|\boldsymbol{z}\| \gg \|\hat{\boldsymbol{z}}\|$. Thus the nodal displacements are dominated by the modes of infinitesimal rigid motion. This allows one to propose the use of these modes as the reduced basis for analysis of under-constrained structures.

To summarize the method proposed, Eqs. (4) and (7), with small strain approximations and appropriate initial and boundary conditions, are solved using a reduced basis consisting of infinitesimal rigid body modes. The efficiency of the proposed reduced basis is further investigated next in "Numerical Simulation." It is worth mentioning that different proposals of reduced bases can be found in the literature (Kirsch 1991; Noor 1994).

Numerical Simulation

Description of Examples

Three examples, shown in Figs. 1–3, are considered. All structures comprise members of circular cross section with radii of 0.2 cm and elasticity module of 2.1×10^6 kg/cm^2.

The first structure is a two-member cable net (Fig. 1) with only one nodal degree of freedom (u). Both members are initially prestressed to 100 kg.

The second structure is a plane cable net (Fig. 2) consisting of 11 members and possessing 12 degrees of freedom. Its degree of kinematic indeterminacy (the difference between the number of degrees of freedom and the rank of the linearized kinematic matrix $\boldsymbol{B}_0$) is 2. This number is also the number of reduced basis modes in accordance with the proposed approach. Prestressing forces in kg take the following form taking symmetry into account: $p_{01}=33$, $p_{02}=29.1682$, $p_{09}=9.90404$, and $p_{010}=14.1586$.

The third structure is a space dome (Fig. 3) comprising 8 vertical struts and 28 cables. It possesses 48 degrees of freedom. Its degree of kinematic indeterminacy is 13. Prestressing forces in kg take the following values taking symmetry into account: $p_{01}=p_{02}=69.282$, $p_{03}=p_{04}=34.641$, $-p_{05}=p_{030}=p_{032}=20$, and $p_{031}=-p_{034}=40$.

It is assumed that all structures possess lumped masses at the nodes and the mass matrices are diagonal unit matrices.

Simulation Techniques

The Mathematica NDSolve procedure (Wolfram 1991) is used for numerical solution of the initial value problem described above. This procedure uses the Adams predictor–corrector method for nonstiff differential equations and backward difference formulas (Gear method) for stiff differential equations. It switches between the two methods using heuristics based on the adaptation of a selected step size. It starts with the nonstiff method under essentially all conditions, and checks the advisability of switching methods every 10 or 20 steps. The algorithms and the heuristics for switching between algorithms were described by Hindmarsh (1983) and by Petzold (1983).

The shooting method is adopted in combination with the IVP solver for solution of the two-point boundary value problem for periodic motions. Assuming that the problem is autonomous, that is, $\boldsymbol{q}=0$, it is possible to reformulate it as follows:

$$\boldsymbol{u}'' = T^2 \boldsymbol{M}^{-1} \boldsymbol{r}[\boldsymbol{u}]$$
$$T' = 0 \qquad (24)$$

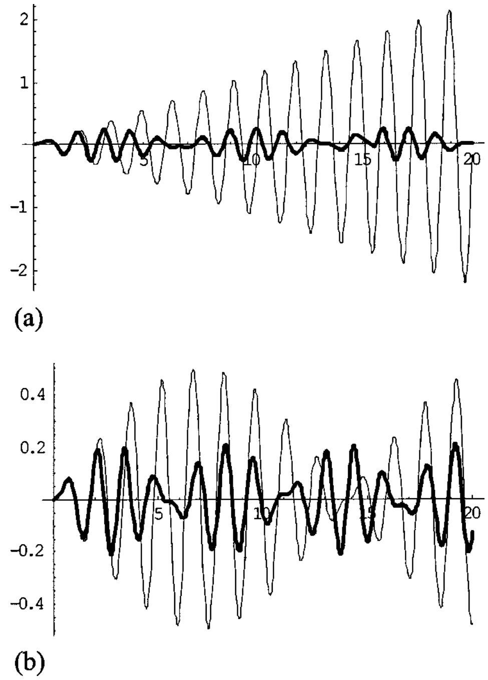

Fig. 4. Displacement versus time for linear resonance (a) and beating (b) of the two-member net (Fig. 1). Fine and bold curves show results of linear and nonlinear analyses, respectively.

and

$$\boldsymbol{u}[0] = \boldsymbol{u}[1]$$
$$\boldsymbol{u}'[0] = \boldsymbol{u}'[1] \qquad (25)$$
$$T[0] = T[1]$$

The prime in Eqs. (24) and (25) refers to normalized time $\tau \in [0,1]$. By introducing new unknowns,

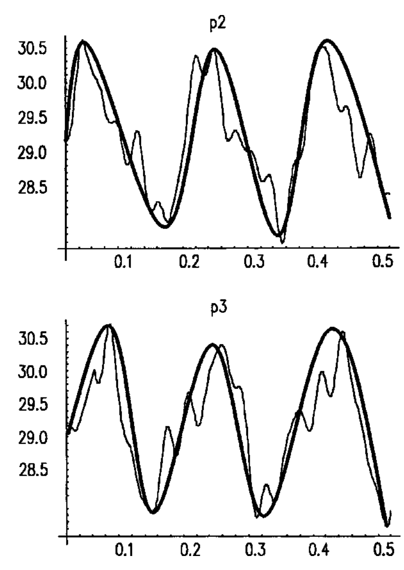

Fig. 5. Axial forces in members 2 and 3 of the plane cable net versus time for free nonlinear vibrations under initial conditions $u_i=0$; $\dot{u}_i=0.5$. Bold curves show results obtained using reduced basis. Bold curves are solutions of 2 nonlinear equations; fine curves are solutions of 12 nonlinear equations.

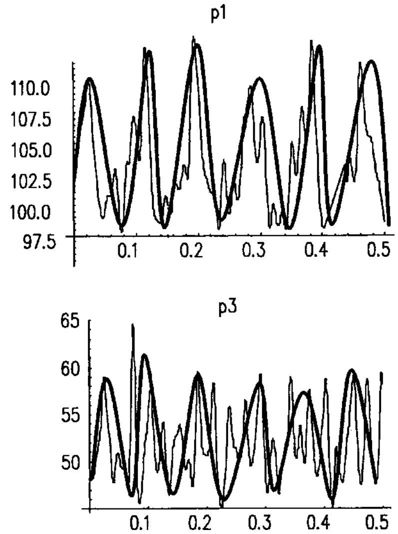

Fig. 6. Axial forces in members 1 and 3 (the worst case) of the space cable dome versus time for free nonlinear vibrations under initial conditions $u_i=0$; $\dot{u}_i=0.5$. Bold curves show results obtained using reduced basis. Bold curves are solutions of 13 nonlinear equations; fine curves are solutions of 48 nonlinear equations.

$$\boldsymbol{y}=\left\{\begin{array}{c}\boldsymbol{u}\\ \boldsymbol{u}'\\ T\end{array}\right\} \tag{26}$$

the problem can be represented in canonical form

$$\boldsymbol{y}'=\boldsymbol{f}[\boldsymbol{y}] \tag{27}$$

$$\boldsymbol{E}_1\boldsymbol{y}[0]+\boldsymbol{E}_2\boldsymbol{y}[1]=\boldsymbol{e} \tag{28}$$

Keller (1992) suggested the following iterative shooting scheme for the solution of Eqs. (27) and (28):

$$\boldsymbol{s}^{(i+1)}=\boldsymbol{s}^{(i)}-\boldsymbol{E}_3(\boldsymbol{E}_1\boldsymbol{s}^{(i)}+\boldsymbol{E}_2\boldsymbol{y}[\boldsymbol{s}^{(i)},1]-\boldsymbol{e}) \tag{29}$$

where $\boldsymbol{s}^{(0)}=\boldsymbol{y}[0]$ is the initial guess and $\boldsymbol{E}_3=(\boldsymbol{E}_1+\boldsymbol{E}_2)^{-1}$. Since, in our case, $\boldsymbol{E}_1=-\boldsymbol{E}_2$ is a unity matrix (and $\boldsymbol{e}$ is a zero vector), matrix $\boldsymbol{E}_3$ should be modified. Specifically, the following diagonal matrix $\boldsymbol{E}_3=\mathrm{diag}\{1/2, 1/2,...,1/2, 1\}$ is used.

The convergence criterion of the procedure (29) was defined as

$$\frac{(\boldsymbol{s}^{(i+1)}-\boldsymbol{s}^{(i)})^T(\boldsymbol{s}^{(i+1)}-\boldsymbol{s}^{(i)})}{(\boldsymbol{s}^{(i)})^T(\boldsymbol{s}^{(i)})}\leqslant \mathrm{tol} \tag{30}$$

Results

A comparison of time histories of forced vibrations of the two-member cable net (Fig. 1) are shown in Fig. 4. Fig. 4(a) presents the linear resonance when the structure is loaded by unit sinusoidal force with critical frequency ω_c, which is equal to the natural frequency of the structure. As expected, a divergent curve appears. However, taking into account geometric nonlinearity the resonance behavior disappears (bold line). Even in Fig. 4(b) with near resonance loading, where $\omega=0.9\omega_c$, the linear approach exhibits beatings (Timoshenko et al. 1974) while the nonlinear behavior (bold line) is still different both qualitatively and quantitatively.

Typical time histories of nonlinear free vibrations that consider axial forces in members 2 and 3 of the plane cable net and axial forces in members 1 and 3 of the space cable dome are shown in Figs. 5 and 6, respectively. Bold lines represent the solution obtained by reduced bases. Fine lines represent the exact solution obtained without reduction of the basis. The reduction in the number of equations is from 12 to 2 in the case of the net and from 48 to 13 in the case of the dome.

Convergence of the shooting procedure considering the periodic solution of free nonlinear vibrations of the plane net is shown in Fig. 7. The final period is $T=0.39$ s. The bold lines represent the solution obtained using reduced bases. Fine lines

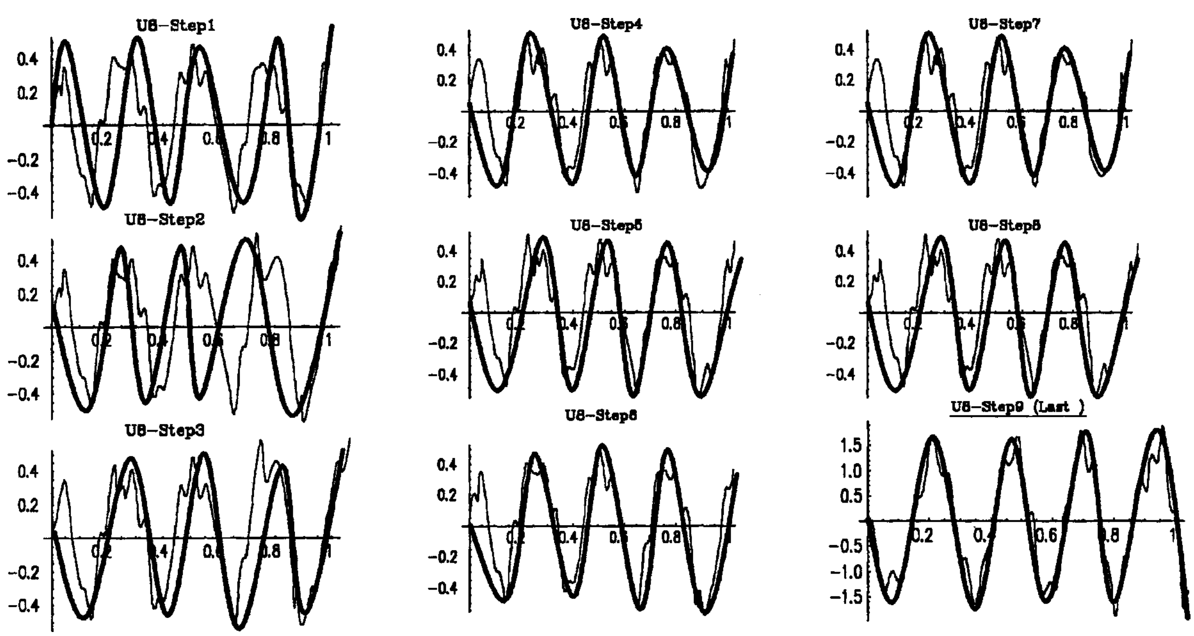

Fig. 7. Convergence to the periodic solution of the plane cable net under conditions $u_i=0$; $\dot{u}_i=1$; $T=0.5$; tol$=10^{-3}$. Bold curves show results obtained using reduced basis; fine curves represent the exact solution obtained without reduction of the basis.

represent the exact solution obtained without reduction of the basis.

The results show that an average of the reduced basis approximations gives exact results. The accuracy of calculations of member forces is unexpected and deserves special attention. It will be discussed further later.

Discussion

Two issues concerning the dynamic response of cable structures where addressed in the numerical simulations. First, the inadequacy of the linear analysis of the dynamic response of cable structures was underscored. The time histories of forced vibrations of the simplest cable net exhibit qualitative and quantitative differences in the results of linear and nonlinear analyses (Fig. 4). Moreover, neither resonance nor beating predicted by the linear analysis was observed in the nonlinear computations. Second, simplified nonlinear models of cable structures were introduced by means of the reduced basis technique. The initial modes of infinitesimal rigid body motion were proposed to be the dominant modes. This simplification, which allows significant reduction of the size of the problem, has been extensively examined in examples of the initial value and two-point boundary value problems. The results of the numerical simulations verify the applicability of the proposed approach. It should be mentioned that the effect of reduced basis is an averaging effect. It extracts the slow component from the dynamic response. This component is usually of major practical interest. Full-scale analysis should be performed if one needs the fast component of motion, which is superimposed on the slow component. It is worth stressing that the reduced basis provides good averaging of the internal forces shown in Figs. 5 and 6. At first glance this is contrary to intuition because the initial infinitesimal modes of rigid body motion do not produce internal force. The following explanation to this ''contradiction'' is provided. The modes of rigid body motion obtained from the linearized equations do not produce member forces when they are *infinitesimal*. Since they are dominant in the overall motion, where displacements are not infinitesimal, the forces produced by them also dominate over the forces produced by ''elastic'' displacements.

Conclusions

It was shown that dynamic analysis of cable structures based on the linear approach is inaccurate both qualitatively and quantitatively. It is necessary to perform geometrically nonlinear dynamic analysis in order to adequately design cable structures. The general dynamic formulation presented in this work provides a computational framework for the analysis of time history (the initial value problem) and periodic motion (the two-point boundary value problem). The most important feature of this method is the reduction of the general set of possible displacement modes to a smaller set of modes called the *reduced basis*. This basis consists of infinitesimal rigid body motion of the unstressed cable structure and it is the right null space of the linearized kinematic matrix. In this way slow motions are extracted from the overall dynamic response of cable structures, which is an averaging process. The numerical advantages of using the proposed basis are the reduction of the size of the problem, which is important in nonlinear computations, and smoothing of the numerical solutions to avoid stiff numerics. The results of numerical simulations presented in this paper favor the use of the computational framework proposed.

References

Argyris, J., and Mlejnek, H. P. (1991). *Dynamics of structures*, North-Holland, Amsterdam.

Buchholdt, N. A. (1985). *Introduction to cable roof structures*, Cambridge University Press, Cambridge.

Crisfield, M. A. (1991). *Non-linear finite element analysis of solids and structures*, Wiley, New York.

Fried, I. (1982). ''Large deformation static and dynamic finite element analysis of extensible cables.'' *Comput. Struct.*, 15, 315–319.

Hindmarsh, A. C. (1983). ''ODEPACK: A systematized collection of ODE solvers.'' *Scientific computing*, R. S. Stepelman, ed., North-Holland, Amsterdam.

Hu, H. Y., and Jin, D. P. (2001). ''Non-linear dynamics of a suspended traveling cable subject to transverse fluid excitation.'' *J. Sound Vib.*, 239(3), 515–529.

Irvine, H. M. (1981). *Cable structures*, MIT Press, Cambridge.

Kahla, N. B. (1995). ''Dynamics of a single guy cable.'' *Comput. Struct.*, 54(6), 1197–1211.

Keller, H. B. (1992). *Numerical methods for two-point boundary value problems*, Dover, New York.

Kirsch, U. (1991). ''Reduced basis approximations of structural displacements for optimal design.'' *AIAA J.*, 29(10), 1751–1758.

Koh, C. G., Zhang, Y., and Quek, S. T. (1999). ''Low-tension cable dynamics: Numerical and experimental studies.'' *J. Eng. Mech.*, 125(3), 347–354.

Krishna, P. (1978). *Cable-suspended roofs*, McGraw-Hill, New York.

Mesarovic, S., and Gasparini, D. A. (1992a). ''Dynamic behavior of nonlinear cable system. I.'' *J. Eng. Mech.*, 118(5), 890–903.

Mesarovic, S., and Gasparini, D. A. (1992b). ''Dynamic behavior of nonlinear cable system. II.'' *J. Eng. Mech.*, 118(5), 904–920.

Milinazzo, F., Wilinazzo, M., and Latchman, S. A. (1987). ''An efficient algorithm for simulating the dynamics of towed cable systems.'' *Ocean Eng.*, 14(6), 513–526.

Nayfeh, A. H., and Balachandran, B. (1995). *Applied non-linear dynamics*, Wiley, New York.

Noor, A. K. (1994). ''Recent advances in applications of reduction methods.'' *Appl. Mech. Rev.*, 47, 125–146.

Perkins, N. C., and Mote, C. D., Jr. (1987). ''Three-dimensional vibration of traveling elastic cables.'' *J. Sound Vib.*, 114, 325–340.

Petzold, L. R. (1983). ''Automatic selection of methods for solving stiff and non-stiff systems of ordinary differential equations.'' *SIAM J. Sci. Comput. (USA)*, 4, 136–148.

Seydel, R. (1994). *Practical bifurcation and stability analysis*, Springer, New York.

Szabo, J., and Kollar, L. (1984). *Structural design of cable suspended roofs*, Horwood, Chichester, U.K.

Timoshenko, S. P., Young, D. H., and Weaver, W. (1974). *Vibration problems in engineering*, 4th Ed., Wiley, New York.

Triantafyllou, M. S., and Howell, C. T. (1992). ''Nonlinear impulsive motions of low-tension cables.'' *J. Eng. Mech.*, 118(4), 807–830.

Volokh, K. Yu. (1999). ''Non-linear analysis of underconstrained structures.'' *Int. J. Solids Struct.*, 36, 2175–2187.

Volokh, K. Yu. (2001). ''Nonlinear analysis of pin-jointed assemblies with buckling and unilateral members.'' *Comp. Model. Eng. Sci.*, 2(3), 389–400.

Volokh, K. Yu., and Vilnay, O. (1997). '' 'Natural,' 'kinematic' and 'elastic' displacements of underconstrained structures.'' *Int. J. Solids Struct.*, 34, 911–930.

Wolfram, S. (1991). *Mathematica: A system for doing mathematics by computer*, 2nd Ed., Addison-Wesley, New York.