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Short communication

Comparison of biomechanical failure criteria for abdominal aortic aneurysm

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ABSTRACT

Medical doctors consider a surgery option for the expanding abdominal aortic aneurysm (AAA) when its maximum diameter reaches 5.5 cm. This simple geometrical criterion may possibly underestimate the risks of rupture of small aneurysms as well as overestimate the risks of rupture of large aneurysms. Biomechanical criteria of the AAA failure are desired. Various local criteria of the AAA failure are used in the literature though their experimental validation is needed. In the present work, we use the experimentally calibrated AAA model, which includes a failure description, to examine various popular criteria of the local failure. Particularly, we analyze various states of the biaxial tension of the AAA material and evaluate the following criteria of the local failure: (1) the maximum principal stretch; (2) the maximum principal stress; (3) the maximum shear stress; (4) von Mises stress; and (5) the strain energy. The results show that the strain energy is almost constant for the failure states induced by the loads varying from the uniaxial to the equal biaxial tension. The von Mises stress exhibits a wider range of scattering as compared to the strain energy. The maximum stresses and stretches vary significantly with the variation of loads from the uniaxial to the equal biaxial tension.

1. Introduction

Bengtsson et al. (1996) and Ouriel et al. (1992) suggest that AAA is found in two percent of the elderly population with more than 150,000 new cases diagnosed each year. The gradually expanding aneurysm ruptures causing death in about 90% of cases. It is the thirteenth most common cause of death in the United States according to Patel et al. (1995). Since the treatment of AAA is expensive and risky, it is very important to predict when the danger of rupture justifies surgery.

Medical doctors consider a surgery option for the expanding abdominal aortic aneurysm (AAA) when its maximum diameter reaches 5.5 cm. This simple geometrical criterion may possibly underestimate the risks of rupture of small aneurysms as well as overestimate the risks of rupture of large aneurysms. Biomechanical criteria of the AAA failure are desired. Various local criteria of the AAA failure are used in the literature though their experimental validation is needed: Elger et al. (1996), Watton et al. (2004), Li and Kleinstreuer (2005), Raghavan and Vorp (2000), Vorp (2007). In the present work, we use the experimentally calibrated AAA model, which includes a failure description, to examine various popular criteria of the local failure. Particularly, we analyze various states of the biaxial tension of the AAA material and evaluate the following criteria of the local failure: (1) the maximum principal stretch; (2) the maximum principal stress; (3) the maximum shear stress; (4) von Mises stress; and (5) the strain energy. The results show that the strain energy is almost constant for the failure states induced by the loads varying from the uniaxial to the equal biaxial tension. The von Mises stress exhibits a wider range of scattering as compared to the strain energy. The maximum stresses and stretches vary significantly with the variation of loads from the uniaxial to the equal biaxial tension.

2. Methods

Volokh and Vorp (2008) proposed a new constitutive theory of AAA assuming that its material is homogeneous, isotropic, and incompressible. They set the strain energy density in the form

$$\psi=\Phi\left\{1-\exp\left[-\frac{\alpha_1}{\Phi}(\mathrm{tr}\,\boldsymbol{C}-3)-\frac{\alpha_2}{\Phi}(\mathrm{tr}\,\boldsymbol{C}-3)^2\right]\right\} \quad (1)$$

where $\boldsymbol{C}=\boldsymbol{F}^T\boldsymbol{F}$ is the right Cauchy–Green tensor; $\boldsymbol{F}$ is the deformation gradient; α_1 and α_2 are the elastic constants (shear moduli) of the material; and Φ is a failure constant—the energy of the molecule separation within a representative volume.

Strain energy (1) was calibrated in the uniaxial tension test (Volokh and Vorp, 2008)

$$\alpha_1=10.3\,\mathrm{N/cm^2};\ \alpha_2=18.0\,\mathrm{N/cm^2};\ \Phi=40.2\,\mathrm{N/cm^2}. \quad (2)$$

Typically, AAA experiences a biaxial stress–strain state in vivo. To mimic it we consider a biaxial deformation in plane ($x_1 x_2$) by using principal stretches, λ_i, and stresses, σ_i,

$$\sigma_i=\lambda_i\frac{\partial\psi}{\partial\lambda_i}-p,\ (\text{no sum over } i). \quad (3)$$

Ignoring the out-of-plane stress, it is possible to find the Lagrange multiplier, p, and the final expression for the Cauchy principal stress

$$\sigma_1=\lambda_1\frac{\partial\psi}{\partial\lambda_1}-\lambda_3\frac{\partial\psi}{\partial\lambda_3},\ \sigma_2=\lambda_2\frac{\partial\psi}{\partial\lambda_2}-\lambda_3\frac{\partial\psi}{\partial\lambda_3},\ \lambda_3=\frac{1}{\lambda_1\lambda_2}. \quad (4)$$

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It is convenient to introduce the biaxiality ratio, n, as follows: $\lambda_1=\lambda$, $\lambda_2=\lambda^n$, $\lambda_3=\lambda^{-(n+1)}$. Thus, we have the uniaxial tension for $n=-0.5$; the equal biaxial tension for $n=1.0$; and the pure shear for $n=0.0$.

3. Results

We analyze the AAA failure in the state of plane stress with varying biaxility, n, for the introduced material model (1). Critical stretches and stresses corresponding to the onset of static instability—failure—are presented in Figs. 1 and 2 accordingly.

Remarkably, the magnitudes of the critical stretches and stresses decrease when the stretch state drifts from the uniaxial tension to the equal biaxial tension. The latter means that the popular failure criteria of the maximum principal stretch or stress are arguable. Indeed, the critical values of the stretch and the stress are usually calibrated in uniaxial tests while Figs. 1 and 2 show that the development of the equal biaxiality of deformation will lead to the failure states with significantly lower magnitudes of the critical stretches and stresses than the ones calibrated in experiments.

Another popular criterion is the maximum shear stress. Its variation in the critical cases of the plane stress is depicted in

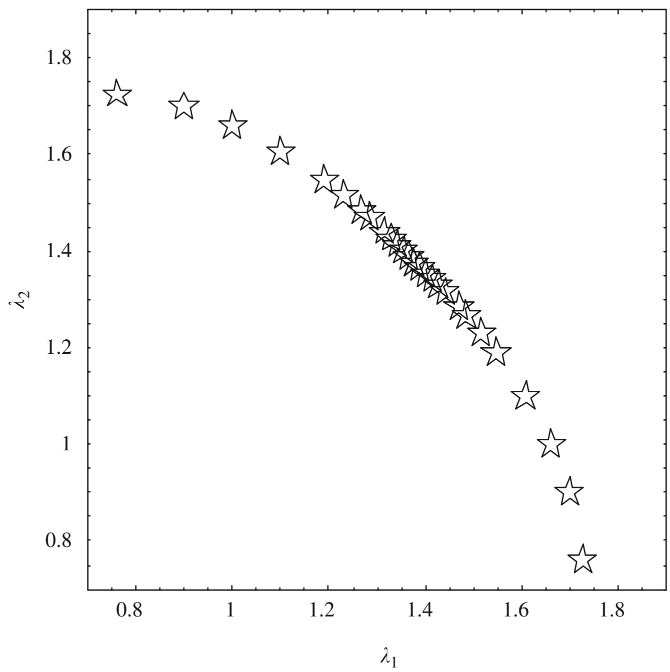

Fig. 1. Critical failure stretches.

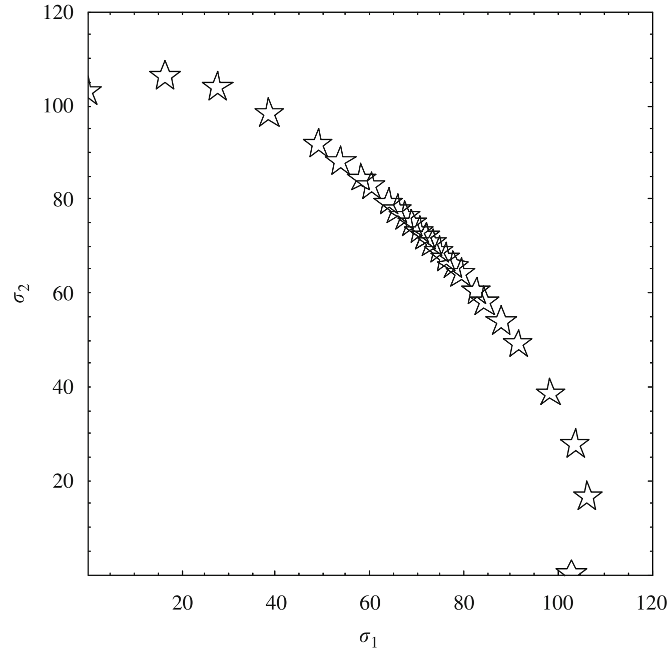

Fig. 2. Critical failure stresses (N/cm^2).

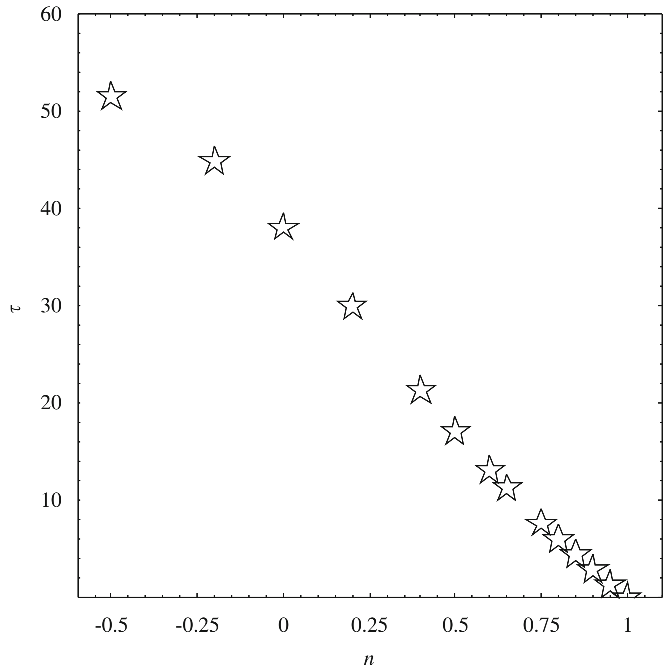

Fig. 3. Critical shear stress, $\tau=(\sigma_1-\sigma_2)/2$, (N/cm^2).

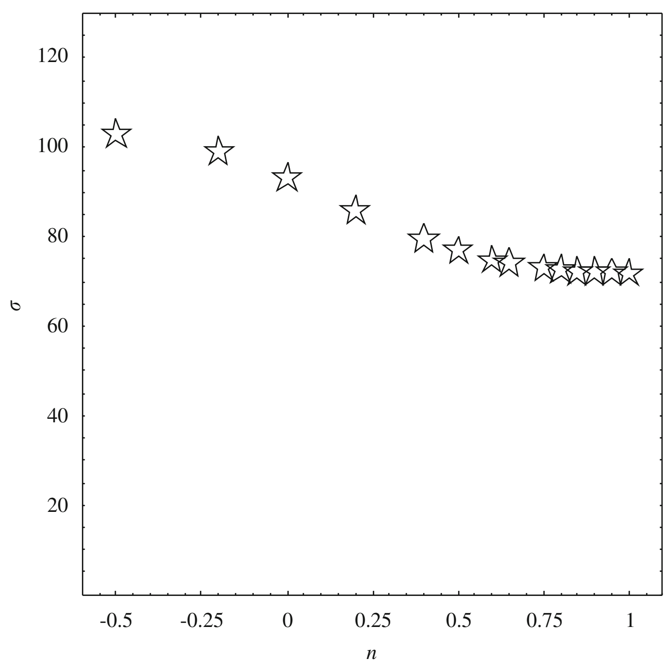

Fig. 4. Critical von Mises stress (N/cm^2).

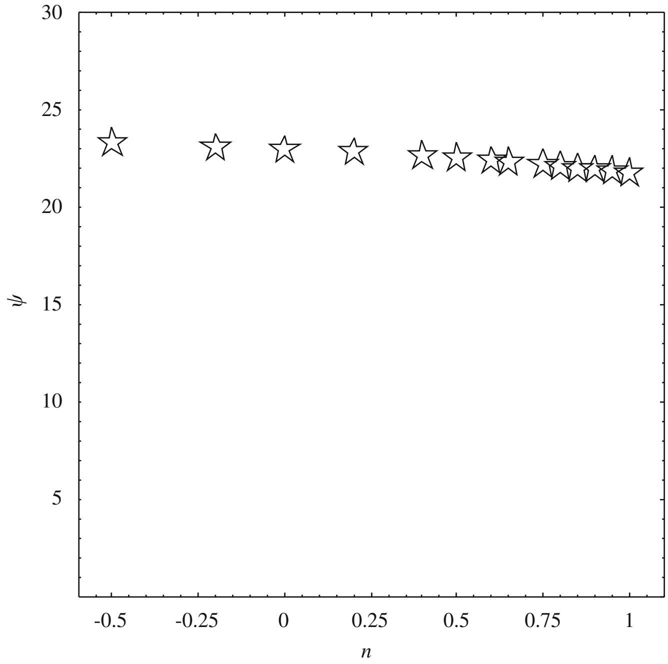

Fig. 5. Critical strain energy (N/cm^2)

Fig. 3. Evidently, the maximum shear stress is not a good choice for the failure criterion by the same reasons as the maximum principal stretches and stresses.

Von Mises true stress, $\sigma=\sqrt{3(\boldsymbol{\sigma}:\boldsymbol{\sigma}-(\mathrm{tr}\boldsymbol{\sigma})^2/3)/2}$, is often used as a failure criterion in order to account for the spatial variations of the stress–strain state. The critical failure magnitudes of the von Mises stress are shown in Fig. 4. The critical values of the von Mises stress calibrated in uniaxial tests where $n=-0.5$ will be somewhat greater than those in the biaxial tension where $n=1$.

Finally, we check the magnitudes of the accumulated strain energy—Fig. 5. The obtained results favor a constant magnitude of the strain energy as a failure criterion. It should not be missed that the critical failure energy, $\psi\approx 22$ N/cm^2, that appears in Fig. 5 does not coincide with the energy of the full separation, $\Phi=40.2$ N/cm^2. The latter means that the material instability leading to rupture starts before the complete separation of all molecules within the representative material volume. The critical failure energy can also be interpreted as the energy corresponding to the limit point on the stress–strain curve where the failure starts while the energy of the full separation corresponds to the point where the stress drops to zero under the increased strain.

4. Discussion

Based on the AAA constitutive model developed in Volokh and Vorp (2008) we analyzed the following criteria of the local material failure in the state of plane stress: (1) the maximum principal stretch; (2) the maximum principal stress; (3) the maximum shear stress; (4) von Mises stress; and (5) the strain energy. We found (Figs. 1–5) that the strain energy is almost constant for the failure states induced by the loads varying from the uniaxial to the equal biaxial tension. The von Mises stress exhibits a wider range of scattering as compared to the strain energy. The maximum stresses and stretches vary significantly with the variation of loads from the uniaxial to the equal biaxial tension.

The obtained theoretical results require further experimental validation in biaxial failure tests. To the best of our knowledge, such experimental data does not exist yet for AAA material. Remarkably, however, the experimental data for the biaxial failure in rubber is available and it favors the conclusions of the present work: Volokh (submitted for publication). We hope that our predictions could encourage the experimenters to shed more light on the issue.

Evidently, AAA material may exhibit anisotropy while the results presented here rely upon the isotropy assumption. The development of simple failure criteria for anisotropic materials will depend on the progress in experimental techniques. Nonetheless, various isotropic models can probably be used as limiting cases for estimating the effect of anisotropy.

Conflict of interest

There is no conflict of interests.

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