Hamilton-Jacobi-Bellman Formalism For Optimal Climate Control of Greenhouse Crops

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***Abstract*— The paper describes a simplified dynamic model of a greenhouse tomato crop, and the optimal control problem related to the seasonal benefit of the grower. A HJB formalism is used and the explicit form of the Krotov-Bellman function is obtained for different growth stages. Simulation results are shown.**

***Index Terms*— HJB equation, Krotov-Bellman function, Greenhouse optimal control.**

I. Introduction

The fast growing sector of The greenhouse horticulture sector is growing fast and is attaining greater economic and social importance. Many efforts have been made to develop advanced computerized greenhouse climate control. In particular, different interesting and important optimal control approaches have been proposed, see e.g. [25], [24], [1], [3], [11], [43], [29], [30], [31], [32], [33], [34], [35], [36], [12], [9], [14], [16].

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However, optimal control concepts and nonlinear dynamic programming (NDP) in particular have almost not been used in practice, due to the complexity of implementation, see [47]. This paper is an attempt to alleviate the complexity problem.

Optimal control theory, see [27], makes it possible to ”transform” a known weather sequence over the growing season to an optimal control sequence, by the simultaneous determination of state and costate variables, and replace the seasonal optimization by instantaneous optimization of the Hamiltonian function at each time step. However, in addition to numerical difficulties, the details of the weather are hard to predict. Here we do not assume a detailed and correct weather forecast, but take an approach based on the so called *climate index*, [42] , and on a simplified crop growth model. Then the Krotov version of nonlinear dynamic programming (NDP), [40], is used *off-line*, and even before the growing season, to determine the scalar *adjusted costate*, [35], which may be interpreted as the optimal control "intensity" [$/kg accumulated dry matter]. By knowing the value of the adjusted costate, the current weather measurement determines the instantaneous optimal control in an on-line optimization procedure, [16]. Clearly, such a procedure is conceptually simple for the grower, and numerically tractable.

The paper is organized as follows: section 2 contains the description of the model and the statement of the problem. Section 3 is devoted to a short description of the Krotov-Bellman sufficient conditions, [40]. Section 4 gives the description of the explicit Krotov-Bellman functions for the three growth stages. Simulation results are reported in Section 5, and the concluding remarks are found in section 6.

II. Description of the simplified model and statement of the problem

The growth of the greenhouse tomato plant is described by two state variables which have different differential equations during different growth stages, [19]. These stages are: vegetative, vegetative-reproductive (mixed), and reproductive. The partition factor that determines the allocation of the accumulated dry matter to vegetative and reproductive parts of the plant is different in each stage.

Many findings, developments and data from [41] are incorporated into this model, which we shall call the MBM-A model. The MBM-A model was calibrated against the data from simulations of the multivariable detailed TOMGRO model, [4], [5]. The state variables of the MBM-A are x and y with given initial conditions

$$x(0) = x_0,\ y(0) = y_0 = 0. \tag{1}$$

Here, x [kg d.m./m^2] is the accumulated vegetative dry mass, d.m., including green leaves, stems and green fruits. The variable y[kg d.m. /m^2] corresponds to the harvestable red fruits (economic end-product). $x_0 > 0$ stands for the seedlings obtained from the nursery. The steady state greenhouse model in [10] gives the function $M(t, U)$[kg/(day*m^2)], that represents the daily rate of dry weight accumulation due to net photosynthesis per unit of sunlit area, and depends on the current outdoor climate inputs at the moment t, and the vector of control variables U, including greenhouse heating, ventilation and CO_2 enrichment. There is a known difference between photosynthesis radiation effectiveness of the reproductive and the vegetative organs, [7], and the dimensionless coefficient k_{rv} represents the ratio between these values of effectiveness.

The dimensionless function $f(x)$ may be loosely defined as the "light interception factor", i.e. the fraction of light intercepted by the canopy. It depends on the leaf area index, L [m^2 leaves/m^2 floor], but can be approximately expressed as a function of the vegetative dry mass, namely,

$$f(x) = 1 - exp(-\beta x),$$

see [35]. The coefficient β and other coefficients were extracted from simulations of TOMGRO. The dimensionless constant value θ represents the loss coefficient of the dry weight allocated to fruits, due to different factors such as fruit abortions, etc.

The determination of the switching time between growth stages is done in terms of τ, *the effective degree-days, EDD*, see [41]. The value of τ is a time integral of $ET[^oC]$,,

$$\tau = \int_{t_0}^{t} ET dt, \qquad (2)$$

where ET is the effective temperature of the greenhouse crop canopy, i.e. the temperature above a given threshold T_l,

$$ET = \max\{0, (T - T_l)\}. \qquad (3)$$

The time $t = t_f$ of the end of the season is fixed. The value $S(t, U)$ [kg d.m./(day m^2)] is equal to $ET\sigma^{-1}$, where

$$\sigma = 10^3[day\ (^oC\ m^2)/(kg\ d.m.)]$$

is a conversion factor due to the given units. Thus S is the effective temperature converted to the units of daily dry matter accumulation. From (2) we have the equation

$$\frac{d\tau}{dt} = S(t, U)\sigma. \qquad (4)$$

Similarly to [42] we assume that the following constant ratio holds,

$$M(t, U)/S(t, U) = K_c. \qquad (5)$$

For open field crops the dimensionless coefficient K_c is a climate index that can be calculated from local climate data history. The mean daily temperature is strongly correlated with the mean daily light, and light is strongly correlated with photosynthesis. Therefore, a strong correlation between temperature and photosynthesis can be assumed, e.g. approximately as in (5). Thus a coefficient K_c, the *climate index*, is an integrated value to characterize the climate. For greenhouse crops this index corresponds to the source/sink activity balance constraint, see [45]. The growth is said to be 'balanced' if the photosynthetic source strength of carbon balances the sink strength of the effectively growing crop. In our greenhouse tomato case, the coefficient K_c can be extracted from TOMGRO, where this proportionality is clearly observed during balanced growth.

The vegetative period is described by the equations

$$\begin{aligned} \frac{dx}{dt} &= M(t, U)f(x), \\ \frac{dy}{dt} &= 0, \end{aligned} \qquad (6)$$

This period starts at $t = t_0$ and ends at $\tau = \tau_1$. However, using (4), (6), and (5) one can see that

$$\frac{dx}{d\tau} = f(x) K_c \sigma. \quad (7)$$

Thus it is easy to calculate the value $x(\tau = \tau_1) = x_1$ which is independent of $U(t)$. Therefore the end of the vegetative period is determined by the moment when $x(t) = x_1$.

In the intermediate (mixed) vegetative-reproductive stage the rate of growth of the red fruits is limited by the potential sink demand of the reproductive organs, and the equations of the process are

$$\frac{dx}{dt} = M(t, U)(1 - \alpha) f(x), \quad (8)$$

$$\frac{dy}{dt} = M(t, U) g(y)/K_c. \quad (9)$$

Equation (8) is an approximation, where the approximation constant α is calibrated such that the vegetative dry matter at the end if the mixed period $x(\tau = \tau_2)$ is equal to the value found in TOMGRO. Equation (9) is equivalent to

$$dy/dt = S(t, U) g(y). \quad (10)$$

Similarly to $f(x)$, $g(y)$ is assumed to be smooth increasing dimensionless function,

$$g(y) = \epsilon + \nu[1 - exp(-\gamma y)].$$

The coefficients $\epsilon,\ \nu,\ \alpha$, all dimensionless, and γ [m^2/kg d.m.], are extracted from TOMGRO simulations. The end of the mixed period at $\tau = \tau_2$, found from TOMGRO, can be restated as the condition $x = x_2$, where x_2 does not depend on the control sequence leading to it. From (8), (9), (4), and (10) we have

$$\frac{dx}{d\tau} = (1 - \alpha) f(x) K_c \sigma$$

$$\frac{dy}{d\tau} = g(y) \sigma. \quad (11)$$

We recall that the values $x(\tau_1) = x_1$ and $y(\tau_1) = y_1 = 0$ are already known, and notice from (11) that the values $x_2 = x(\tau_2)$ and $y(\tau_2) = y_2$ can be easily determined. It will be shown below that $x_2(\tau_2) = x_(t_f)$, thus the value x_2 is a boundary condition for the variable x at the final time t_f.

During the third period (the reproductive stage) all the assimilates are directed to the reproductive organs, and the state equations become

$$\frac{dx}{dt} = 0,$$

$$\frac{dy}{dt} = M(t, U) f(x) \eta. \quad (12)$$

Here the notation is used

$$\eta = k_{rv} \theta \xi. \quad (13)$$

The overall fruit loss coefficient η for the reproductive stage is a product of k_{rv}, θ, and ξ, where the additional coefficient $\xi < 1$ is added to reflect the fact that some photosynthetic assimilates are used to compensate for dying leaves, etc. With this approximation we see that x remains constant, while y is growing linearly.

The reproductive period ends at the given final time $t = t_f$. The final value $x(t_f) = x_2$ is a fixed boundary condition at the end of the trajectory.

The performance criterion (the objective of the optimal control problem) is

$$Q = c_r y(t_f) - \int_{t_o}^{t_f} q(t, U)dt \to \max \quad (14)$$

which represents the maximization of the grower's monetary net income, i.e. the difference between the sales price of the harvestable (red) fruits and the cost of the greenhouse operation, $\int_{t_o}^{t_f} q(t, U)dt$. Here c_r $[\$/(kg\ d.\ m.)]$ is the unit price of red fruits. The cost $q(t, U)$ is determined as

$$q(t, U) = c_h h + c_C C,$$

where h [J/day/m^2]is the heating, and C [kg CO_2/day/m^2] is the CO_2 enrichment control fluxes, respectively, and c_h [\$/J/m^2], c_C [\$/kg/m^2] are the corresponding unit prices. The objective function (14) contains a function of the final state, and an integral part; thus it is a so-called Bolza problem, see [8].

Due to the three different growth periods, one may define different regions in the state space (phase plane) which characterize the solution of the differential equation (6), (8), (9), and (12), illustrated in Figure 1. The regions are:

- *Region 1 (vegetative)*: $x_0 \le x \le x_1$, with the assumption that the initial amount of fruit which remains constant during the vegetative state is limited by $0 \le y \le y^u(x_1)$, where $y^u(x_1)$ is the maximally possible amount of fruit that could have been achieved by the amount of vegetative parts at the end of the vegetative period. Normally, the initial amount of fruit is zero or negligibly small.
- *Region 2 (mixed)*: $x_1 \le x \le x_2$ and $0 \le y \le y^u(x)$, where $y^u(x)$ is the solution of the right hand sides of (9) and (12) being equal, i.e.

$$\eta f(x) = \frac{g(y)}{K_c}. \quad (15)$$

  The mixed period ends, when $y = y^u(x)$ which will occur for $x = x_2$ for the optimal trajectory. See Figure 1.
- *Region 3 (reproductive)*: $x_1 \le x \le x_2$ and $y \ge y^u(x)$. In the reproductive period, the sink demand of the fruit is limited by the available source of photosynthetic materials.
- *Region 4 (infeasible)* : $x_0 \le x \le x_1$, and $y \ge y^u(x_1)$, corresponds to unrealistic initial conditions.

Note that the border between *Region 2* and *Region 3* is the line $y = y^u(x)$ which is the solution of (15) indicating the conditions when the sink demand of the reproductive organs is limited by the available source of photosynthetic materials.

Figure 1 also shows schematically the optimal trajectory of MBM-A, indicated by the bold blue line.

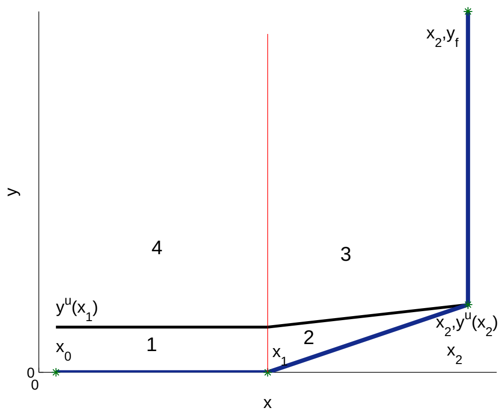

Fig. 1. The characteristic solution regions in the state space of equations (6), (8), (9), and (12). The optimal trajectory is indicated in blue bold.

III. Krotov-Bellman sufficient conditions.

We are using the Hamilton-Jacoby-Bellman formalism in the form of Krotov-Bellman sufficient conditions, see [38], [39], [40]. Krotov's approach is different from Bellman's. Unlike Bellman [2], Krotov considered sufficient conditions for the global optimum. The Krotov function which will be called here the Krotov-Bellman function, coincides with the Bellman function on the optimal trajectory. A comparison of the Krotov and Bellman approaches can be found in [6].

Krotov's sufficient conditions are as follows. For the system of differential equations

$$dz/dt = f(z, u, t) \tag{16}$$

and the objective function

$$G(z, t_f) + \int_{t_0}^{t_f} f_0(z, u, t)dt \to \min \tag{17}$$

the function $R(z, u, t)$ is constructed as

$$R(z, u, t) = V_z(z, t)f(z, u, t)$$
$$-f_0(z, u, t) + V_t(z, t). \tag{18}$$

Here $u \in \overline{U}$, and V_z, V_t are the corresponding partial derivatives of the Lipshitz continuous and piece-wise smooth Krotov-Bellman function $V(x, t)$. Note that the terms in (18) that depend on the control $u(t)$, i.e.

$$H(x, u, t) = V_z(z, t)f(z, u, t) - f_0(z, u, t) \tag{19}$$

is the *Hamiltonian* along the optimal trajectory, to be maximized with respect to u at each time t. The Krotov-Bellman sufficient conditions are

$$\sup_{u\in\overline{U}} R(z, u, t) = \mu(t),$$
$$\phi(z, t_f) = [G(z, t_f) + V(z, t_f)],$$
$$\inf_z \phi(z, t_f) = \phi(z^*(t_f), t_f). \tag{20}$$

Here $z^*(t)$ is an optimal trajectory. It is easy to see that the condition

$$\phi(z, t_f) = const \tag{21}$$

can be used instead of the last equation in (20). Without lack of generality one can set

$$\mu(t) = 0 \tag{22}$$

by the appropriate choice of a function $\zeta(t)$ which will serve as an additive time dependent part of $V(z,t)$. When the system of equations (16) has discontinuous partial derivatives on some manifolds in the ($z,\ t$) space, the Krotov-Bellman function has to be found separately in each region delineated by the manifolds, and then checked for continuity. More details on piece-wise smooth Krotov-Bellman functions are found in [37].

IV. Description of explicit Krotov-Bellman functions

A. Vegetative stage

Our approach to solve analytically the Krotov-Bellman equation at different stages is as follows. We construct the Krotov-Bellman function as a sum of some "basic" functions with unknown coefficients which are later determined from the transversality and continuity conditions on the borders between the different regions.

In the vegetative stage, *Region 1*, we propose

$$V^1(x,y,t) = \int_{x_0}^{x} \frac{p_x^1 dx}{f(x)} + \int_0^y \frac{p_y^1 dy}{g(y)} + \zeta(t), \tag{23}$$

where $p_x^1,\ p_y^1$ are constants. Thus from (6) and (23) we obtain

$$\frac{\partial V^1}{\partial x} f(x) = p_x^1, \tag{24}$$

and the Krotov-Bellman equation (20) is reduced to the equation

$$R(x,y,U,t) = M(t,U)p_x^1 - q(t,U) + \frac{\partial \zeta}{\partial t},$$
$$U^* = arg \sup_U R(x,y,U,t) \tag{25}$$

Let us denote the constant coefficient of $M(t,U)$ as N , i.e. $N = p_x^1$. Referring to equation (19), the control vector $U(t)$ for each moment t is found by maximizing the Hamiltonian

$$NM(U,t) - q(U,t). \tag{26}$$

which depends on the constant N. To explain the significance of N, assume that the maximizing U is found by setting

$$N\frac{\partial M(U,t)}{\partial U} - \frac{\partial q(U,t)}{\partial U} = 0. \tag{27}$$

Then, clearly

$$N = \frac{\partial q(U,t)}{\partial M(U,t)} \left[\frac{\$/m^2/\mathrm{day}}{kg/m^2/\mathrm{day}}\right] \tag{28}$$

and the unit of N is hence $[\$/kg]$. We may therefore interpret N as the *intensity of cultivation* or *intensity of control effort*, i.e. how much the grower pays for a one kg increase of dry matter growth. Below we shall show that N, defined as the coefficient of $M(t,U)$, is constant throughout the growing season.

The function $\zeta(t)$ in (23) can be chosen from the condition

$$R(U^*,t) = 0 \tag{29}$$

by the time integration of the function $\partial\zeta/\partial t$, see (25), (22).

We recall that the vegetative period is bounded by the condition $x_0 \leq x \leq x_1$, and that the conditions $0 \leq y \leq y^u(x_1)$ is satisfied automatically. Let us denote

$$C_1 = \int_{x_0}^{x_1} \frac{p_x^1}{f(x)} dx,$$

and thus the value of the function V at $x = x_1$ can be calculated as

$$V^1(x_1, y, t) = \int_0^y \frac{p_y^1 dy}{g(y)} + C_1 + \zeta(t). \quad (30)$$

B. Vegetative-reproductive stage

The mixed stage, *Region 2*, is determined by the conditions $x_1 \leq x \leq x_2$, and $0 \leq y \leq y^u(x)$. We choose the Krotov-Bellman function as

$$V^2(x, y, t) = \int_0^y \frac{p_y^2 dy}{g(y)} + \zeta(t) + C_1, \quad (31)$$

where p_y^2 is a constant. By assuming

$$p_y^1 = p_y^2 \quad (32)$$

the function $V(x, y)$ is continuous at the border $x = x_1$. From (31), (9), (20), and (18) we have

$$\frac{\partial V}{\partial y}\frac{g(y)}{K_c} = \frac{1}{K_c},$$
$$R(x, y, U, t) = M(t, U)\frac{p_y^2}{K_c}$$
$$-q(t, U) + \frac{\partial \zeta}{\partial t}. \quad (33)$$

We see that for the mixed stage

$$N = p_y^2/K_c. \quad (34)$$

Thus, during the mixed stage we have

$$R(x, y, U, t) = M(t, U)N$$
$$-q(t, U) + \frac{\partial \zeta}{\partial t}, \quad (35)$$

from which U^* is obtained by maximizing (35) over U. By assuming

$$p_y^2/K_c = p_x^1 \quad (36)$$

we get the function $\zeta(t)$, from (29). It is essential that the function $\zeta(t)$ depends only on time t. From (36), and by denoting

$$C_2(x) = \int_0^{y^u(x)} \frac{p_y^2 dy}{g(y)}, \quad (37)$$

we get, at the border $y = y^u(x)$,

$$V^2(x, y^u(x), t) = C_2(x) + \zeta(t) + C_1. \quad (38)$$

C. Reproductive stage

During the reproductive stage, *Region 3*, we let

$$V^3(x, y, t) = C_1 + C_2(x)$$
$$+\frac{p_y^3(y - y^u(x))}{f(x)} + \zeta(t). \quad (39)$$

On the border $y = y^u(x)$ it holds that

$$V^3(x, y^u(x), t) = C_1 + C_2(x) + \zeta(t). \quad (40)$$

From (12) and (39) we get

$$R(x, y, U, t) = M(t, U)p_y^3\eta$$
$$-q(t, U) + \frac{\partial \zeta}{\partial t}. \quad (41)$$

The optimal U^* is found by maximizing R over U. By the assumption

$$p_y^3\eta = N = \frac{p_y^2}{K_c}, \quad (42)$$

and (29), we find that the function $\zeta(t)$ depends on time, only. From (37), (38), and (40) it follows that $V(x, y, t)$ is continuous at the border $y = y^u(x)$.

D. Transversality condition

At the end of the growing season we have the free value $y(t_f)$ and the fixed value $x = x_2$. From the third of equations (20) and (14) we get

$$\phi(x_2, y, t_f) = G(x_2, y, t_f) + V^3(x_2, y, t_f)$$
$$= -c_r y(t_f) + p_y^3 \frac{y(t_f) - y^u(x_2)}{f(x_2)}$$
$$+C_1 + C_2(x_2) + \zeta(t_f). \quad (43)$$

Clearly, the function ϕ is constant if it holds that

$$p_y^3 = \frac{c_r}{f(x_2)}. \quad (44)$$

Hence we can get N from (42), p_y^2 from (34), and p_x^1 from (36). This concludes the solution. We now have all the constants that are needed for the optimal control determination, and all the Krotov-Bellman sufficient conditions are satisfied. We recall that the constant value of N can be seen as the intensity of the seasonal control, or as an *adjusted costate*, see [35].

V. Simulation of the Solution

The optimal value of the constant seasonal control intensity N can be approximately obtained from the MBM-A model above, and then applied to a real greenhouse, or to a comprehensive model, e.g. TOMGRO [4], [5] for online optimization in order to obtain the optimal greenhouse control. The optimal trajectory of x and y [kg d. m./m^2] are shown in Fig. 1, using the TOMGRO and MBM-A models for the numerical example with periodically constant weather from [35]. The optimization algorithm in [46] was used for the determination of the control $U(t)$ at each time step. We have used the data from TOMGRO instead of real experimental data, because TOMGRO has been carefully calibrated with a large experimental data set.

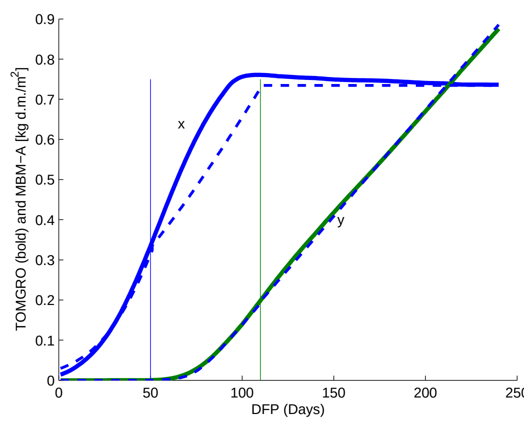

Fig. 2. Seasonal growth, x and y $[kg\ d.m./(day{*}m^2]$: TOMGRO (bold lines) and MBM-A (dashed lines) models. Growth stages indicated. DFP is Day From Planting.

VI. Discussion and Conclusions

A scheme of a seasonal optimal control policy determination based on the Hamilton-Jacoby-Bellman formalism has been presented, and the explicit Krotov-Bellman functions were found for the different growth stages. This approach for the MBM-A model can then be used in conjunction with a comprehensive model like TOMGRO, [4], [5] in order to find the instantaneous on-line control depending on the currently measured weather inputs. We showed how the intensity of control, N, is obtained from the MBM-A model. The TOMGRO model utilized this value by transferring it, together with the daily values of the leaf are index (LAI) and dry weight of the vegetative organs (W), to the GH greenhouse model, [10], together with data of the outdoor weather, [35]. The GH model calculates current values of the control fluxes and the resulting indoor climate, and returned this data to TOMGRO. Thus the overall scheme is rather simple and transparent. Simulation results were shown and compared with the theoretical solution of the MBM-A model. We consider our scheme as a possible and indeed promising basis for future experimental work.

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