ROBUST AND ADAPTIVE CONTROL - FIDELITY OR A FREE RELATIONSHIP?

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Abstract

Robust and adaptive control are essentially meant to solve the same control problem: Given an uncertain LTI model set with the assumption that the controlled plant slowly drifts or occasionally jumps in the allowed model set, find a controller that satisfies the given servo and disturbance rejection specifications. Specifications on the transient response to a sudden plant change or "plant jump" are easily incorporated into the robust control problem, and if a solution is found, the robust control system does indeed exhibit satisfactory transients to plant jumps. The reason to use adaptive control is its ability, when the plant does not jump, to maintain the given specifications with a lower-gain control action (or to achieve tighter specifications), and also to solve the control problem for a larger uncertainty set than a robust controller. Certainly Equivalence based adaptive controllers, however, often exhibit insufficient robustness and unsatisfactory transients to plant jumps. It is therefore suggested in this paper that adaptive control always be built on top of a robust controller in order to marry the advantages of robust and adaptive control. The concept is called Adaptive Robust Control. It may be compared with Gain Scheduling, Two-Time Scale Adaptive Control, Intermittent Adaptive Control, Repeated Auto-Tuning, or Switched Adaptive Control, with the important difference that the control is switched between robust controllers that are based on plant uncertainty sets that take into account not only the currently estimated plant model set but also the possible jumps and drifts that may occur until the earliest next time the controller can be updated.

1. Introduction

Plant uncertainty was always at the heart of feedback control theory. Various robust control methods were developed to handle extended plant uncertainty. Based on classical control, Quantitative Feedback Theory (QFT) was invented, see Horowitz (1993). Based on modern control theory, $\boldsymbol{H}_\infty$, $\boldsymbol{H}_2$, and the μ-methods, were developed, see Sánchez-Peña and Sznaier (1998). Robust pole placement is described in Ackermann (1993). All robust methods are also strongly supported by and dependent of design software (e.g. Gutman, 1996).

The robust control problem is in general NP-hard. Still, the available computational tools have proved to be very useful. If successful, every robust design results in an LTI controller that controls any "frozen" plant in the plant uncertainty set, satisfying the given specifications. It is not possible to tell, *a priori*, if a given robust control problem has a solution without performing (a part of) the design.

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Adaptive control emerged as an alternative to handle uncertain plants. The idea is to combine an on-line identification algorithm with a control design method to yield a controller that follows the changing plant, see Åström and Wittenmark (1995). In spite of their obvious conceptual appeal, and an impressive development effort, adaptive controllers, in particular those based on the certainty equivalence principle, have not become as ubiquitous in industry as expected. The reason for this seems to be that closed loop stability cannot be guaranteed on the same level of confidence as with linear controllers, that adaptive controllers often seem to have unsatisfactory transient behaviour during adaptation to a plant change (e.g. during a start-up when the adaptive controller is initially wrongly tuned), and that they demand highly skilled and educated personnel for tuning and maintenance.

This paper is not meant to be a review of or a comparison between various robust and adaptive control design methods. Instead we try to view adaptive control from a robust point of view, and suggest a remedy for some of their respective shortcomings by marrying them to each other in a suitable way. Because of its graphical nature, QFT is used as a tool to illustrate the concepts.

The paper is organized as follows: In section 2 the control problem is defined. Sections 3 and 4 contain very brief descriptions of robust control and adaptive control, respectively. In section 5 an illustrating example is given of the transient behaviours after a sudden plant change of a robust and adaptive control system, respectively with a summary of their advantages and disadvantages. The argument is made clear in section 6 where adaptive control is seen from a robust perspective. The rôle of adaptation is discussed in section 7, leading up to the suggested paradigm of Adaptive Robust Control in section 8. A short conclusion is found in section 9, followed by some references.

2. Problem definition

For simplicity, we consider the SISO case only. Given an uncertain, strictly proper LTI plant

$$\boldsymbol{P}(\boldsymbol{s}) \in \{\boldsymbol{P}_i(\boldsymbol{s})\} = \frac{\boldsymbol{n}(\boldsymbol{s},\boldsymbol{p})}{\boldsymbol{d}(\boldsymbol{s},\boldsymbol{p})} e^{-p_q s}\left(\boldsymbol{1}+\boldsymbol{M}(\boldsymbol{s})\right) \tag{1}$$

with the uncertain parameter vector $\boldsymbol{p} \in \Pi \in \Re^{\boldsymbol{q}}$, and the multiplicative unstructured uncertainty satisfying $|\boldsymbol{M}(j\omega)| \le \boldsymbol{m}(\omega)$. $\boldsymbol{M}(\boldsymbol{s})$ is assumed to be stable and proper, and the high frequency gain sign of (1) is known. The index *i* in (1) only denotes membership in the set and not enumeration. Input, state, and output disturbances, $\boldsymbol{D}_i(\boldsymbol{s})$, are assumed to act on the plant. The closed loop specifications are given as a servo specification,

$$\boldsymbol{a}(j\omega) \le |\boldsymbol{Y}(j\omega)/\boldsymbol{R}(j\omega)| \le \boldsymbol{b}(j\omega) \tag{2}$$

or sensitivity specification,

$$|\boldsymbol{S}(j\omega)| \le \boldsymbol{x}(\omega) \tag{3}$$

or any other disturbance rejection specification.

It is assumed that the plant P(s) “slowly” drifts among $\{\boldsymbol{P}_i(\boldsymbol{s})\}$. Such a drift may be caused by wear, change of operating point, or a change in the outside environment. Moreover, it is assumed that P(s) “occasionally” jumps within $\{\boldsymbol{P}_i(\boldsymbol{s})\}$, i.e. suddenly changes from one LTI plant instance to another. Such a jump may be caused by a change of plant equipment, or a partial failure, or unknown loading (e.g. when a robot arm picks up an unknown load), or when switching on the control system without knowing which member of the set (1) describes the plant at that moment. It is assumed that the frequency of the jumps is considerably lower than the bandwidth of the closed loop system and that the speed of the drift is considerably lower than e.g. the speed of the step response transient of the closed loop system, see e.g Åström and Wittenmark (1995). One could say that the “product of plant change and time is small” (Goodwin *et al.*, 2000). The control design problem is then to find a controller that makes the closed loop system satisfy the specifications.

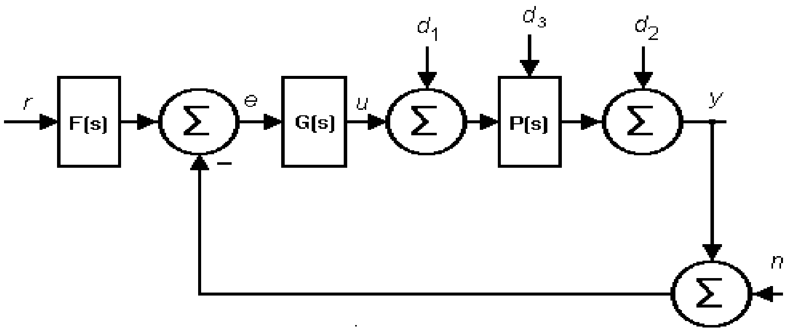


Figure 1: The closed loop control system for robust control.

3. Robust Control

The robust control problem is to find a feedback controller $\boldsymbol{G}(\boldsymbol{s})$ and a prefilter $\boldsymbol{F}(\boldsymbol{s})$ (see Figure 2), such that the specifications (2), (3), etc, are satisfied for each member in the plant set $\{\boldsymbol{P}_i(\boldsymbol{s})\}$. Note that the actual controller implementation may differ from the canonical form shown in Figure 1.

If a solution is found, then the specifications are also satisfied for slow plant drift, due to the quasi-LTI assumption in section 2. If, in addition, the specifications include the rejection of disturbances equivalent to the envisaged "occasional jumps", and a solution is found, then the specifications are also satisfied during plant jumps.

Some robust control methods were mentioned in the introduction. Here a few details about QFT will be given, since QFT will be used for illustration. For the reader familiar with the $\boldsymbol{H}_\infty$-method, we would like to point out that the SISO robust sensitivity problem for a plant with unstructured multiplicative uncertainty only, is identical for $\boldsymbol{H}_\infty$ and for QFT (cf. Figure 2.17 in Sánchez-Peña and Sznaier, 1998).

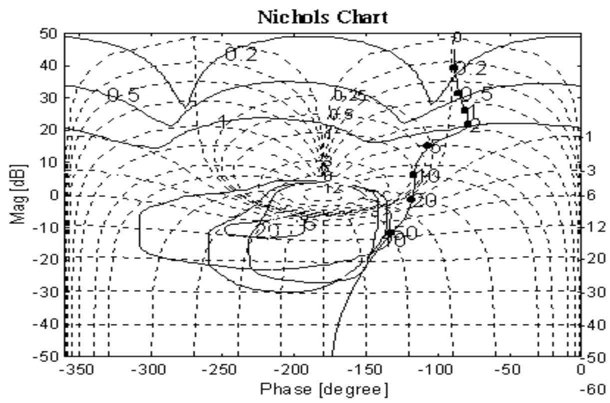


Figure 2: The Horowitz bounds $\partial \boldsymbol{B}_L(j\omega)$ for some frequencies, together with the nominal open loop, $\boldsymbol{L}_{\mathbf{nom}}(j\omega)$, in a Nichols chart.

In QFT, the specifications and the plant uncertainty set $\{\boldsymbol{P}_i(\boldsymbol{s})\}$ gives rise, for each frequency ω, to a set $\boldsymbol{B}_G(j\omega) \in C$ such that $\boldsymbol{G}(j\omega) \in \boldsymbol{B}_G(j\omega) \Leftrightarrow$ the specifications are satisfied for each member in $\{\boldsymbol{P}_i(\boldsymbol{s})\}$. Then, $\boldsymbol{B}_L(j\omega) = \boldsymbol{B}_G(j\omega)\boldsymbol{P}_{\mathbf{nom}}(j\omega)$, where $\boldsymbol{P}_{\mathbf{nom}}(\boldsymbol{s}) \in \{\boldsymbol{P}_i(\boldsymbol{s})\}$ is an arbitrary nominal plant, such that $\boldsymbol{L}_{\mathbf{nom}}(j\omega) \in \boldsymbol{B}_L(j\omega) \Leftrightarrow$ the specifications are satisfied for each member in $\{\boldsymbol{P}_i(\boldsymbol{s})\}$, where $\boldsymbol{L}_{\mathbf{nom}}(\boldsymbol{s}) = \boldsymbol{P}_{\mathbf{nom}}(\boldsymbol{s})\boldsymbol{G}(\boldsymbol{s})$ is the nominal open loop. In general, the *Horowitz bound* $\partial \boldsymbol{B}_L(j\omega)$ is displayed in a Nichols chart. With Horowitz bounds for several frequencies displayed, $\boldsymbol{L}_{\mathbf{nom}}(j\omega)$ is manually loopshaped in order to satisfy the specifications. See Figure 2.

4. Adaptive Control

The ideal adaptive control would be *dual control* (Åström and Wittenmark, 1995), in which the control signal is optimal for both plant estimation and control. Unfortunately dual control is computationally prohibitive. LTI based adaptive control should have the following desired capabilities: estimate the current plant model, redesign the controller, an decide when to estimate/redesign (Goodwin *et al*., 2000). Many types of adaptive controllers do not have all these features; instead, a practical definition of adaptive control could be: "an adaptive controller is a controller with adjustable parameters, and a mechanism for adjustment" (Åström and Wittenmark, 1995). See Figure 3. The parameters are adjusted such that after convergence, the specifications (2), (3), etc, are satisfied.

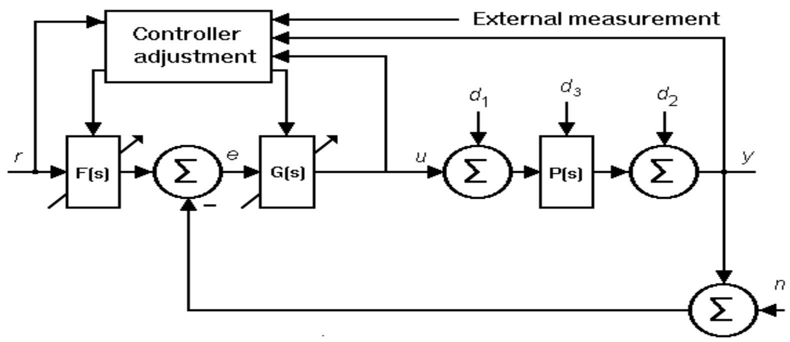


Figure 3: The closed loop control system for adaptive control.

The *Certainty Equivalence* (CE) adaptive controller combines parameter estimation (RLS, LMS, etc) with some control design method (pole placement, MRAS, etc). The controller parameter are computed as if the current parameter estimate is true. Under ideal conditions (no noise, no disturbances, no undermodelling, minimum phase plant) MRAS gives boundedness and convergence (Åström and Wittenmark, 1995). To handle some of the non-ideal conditions, detuning from CE, such as dead-zone, σ-modification, back-stepping, etc, have been suggested (Goodwin *et al*., 2000). A functioning CE adaptive controller handles slow plant drift very well. The adaptation transient after the occasional plant jump is however often very unsatisfactory, as illustrated in the next section.

An *auto-tuner* (Åström and Wittenmark, 1995) is an adaptive controller where the control law is automatically updated only after the convergence of the parameter estimate, and only on human operator demand and under operator supervision. Thus the disadvantages of the CE adaptive controller are avoided, but an auto-tuner is not able to handle neither plant drift nor plant jumps without human intervention.

Gain scheduling (Åström and Wittenmark, 1995) is a scheme where you apply different pre-computed control for each operating condition. The operating condition is given in a separate identification loop based on measured external signals or process variables. Often soft transfer between the different controllers is implemented. Since the system is almost LTI at all times, there are no stability problems. Gain scheduling handles plant drift well, if for each operating condition the controller is robust. Plant jumps within an operating condition are also handled if the local controller is designed appropriately.

The difficulties with CE adaptive control seems to be due to the interference between the identification and control loops (Goodwin *et al*., 2000). Therefore a new type of adaptive controllers have been suggested and researched lately, under names such as Two time scale adaptive control, Intermittent adaptive control, or Switched adaptive control, attempting to combine the advantages of Auto-tuning and Gain Scheduling. The identification and control loops are separated with the control parameters being updated only when necessary and when the identification has converged. Hence the closed loop is LTI at (almost) all times and there is no local stability problem. It is however not clear how these controllers behave during slow plant drift and occasional plant jumps. This is a main issue of this paper.

5. Robust vs. Adaptive

This example is found in Åström *et al.* (1986). Consider the uncertain plant

$$P(s)=\frac{k}{(1+Ts)^2} \quad \text{with} \quad k\in[1,4] \quad \text{and} \quad T\in[0.5,2]. \tag{4}$$

A QFT design was performed with a servo specification having a bandwidth of 2 rad/s. Simulation results are shown in Figure 4. The resulting controller was

$$G(s)=4\cdot 10^7\cdot\frac{s+0.25}{s}\cdot\frac{s+1.5}{s+30}\cdot\frac{1}{s^2+500s+250000} \quad \text{and} \quad F(s)=2.89\cdot\frac{1}{s^2+1.87s+2.89}. \tag{5}$$

As a comparison, an explicit second order adaptive pole placement controller, with RLS as the parameter estimator was implemented with a sampling interval = 0.3 seconds. The required pole location had a natural frequency = 1.5 rad/s and the relative damping = 0.707. Thus the specifications of the two designs were similar. Simulations of the adaptive control system are found in Figure 5.

It is clear from the simulations that the adaptive controller exhibits unsatisfactory transients during adaptation, while the robust controller works fine. However, after convergence, the adaptive controller has the same reference step response for different plant cases. The same conclusion is drawn from Example 10.1 in (Åström and Wittenmark, 1995).

Robust control is stable and satisfactorily controls the plant during occasional plant jumps and slow plant drift. A CE adaptive controller is stable only under restrictive assumptions, exhibits ugly transients during

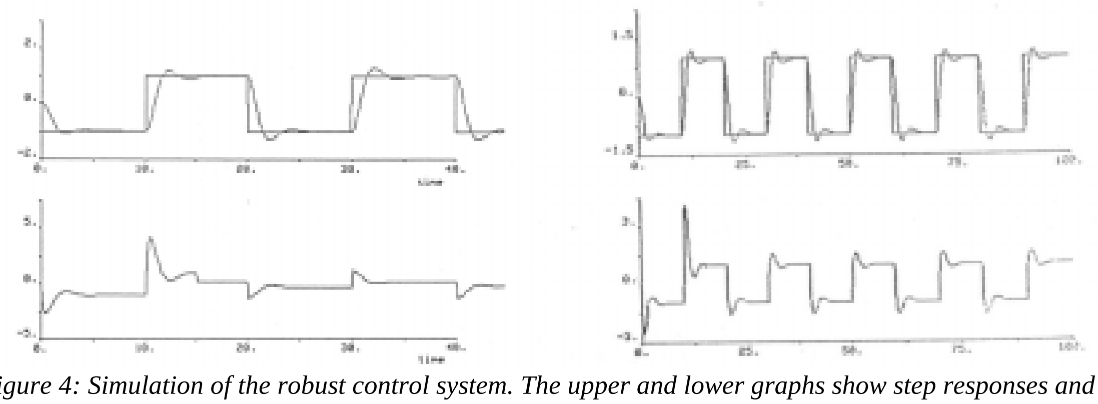


*Figure 4: Simulation of the robust control system. The upper and lower graphs show step responses and the control signals, respectively. On the left, the plant gain changes from 1 to 4 at time=15 s while **T**=1. On the right, the time constant **T** changes from 1 to 0.5 at time t=15 s while **k**=1.*

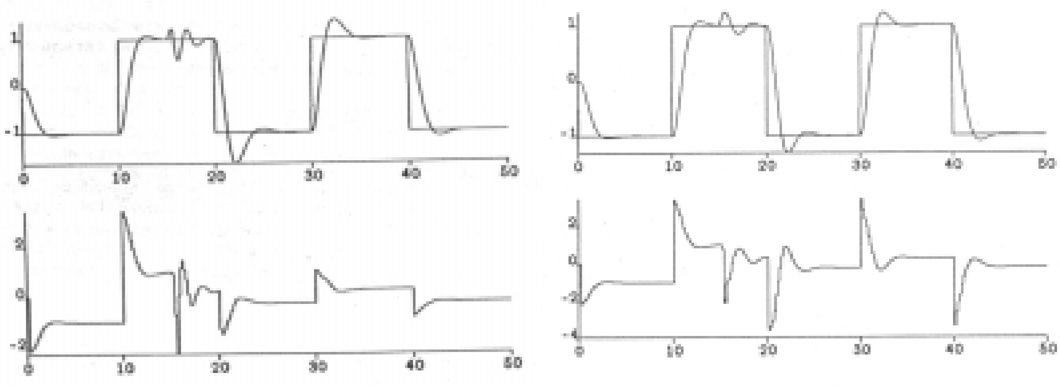


*Figure 5: Simulation of the adaptive control system. The upper and lower graphs show step responses and the control signals, respectively. On the left, the plant gain changes from 1 to 4 at time=15 s while **T**=1. On the right, the time constant **T** changes from 1 to 0.5 at time t=15 s while **k**=1.*

plant jumps, but controls the plant more uniformly during slow or no drift. In fact, an adaptive control solution may be found (with the exception of plant jumps) for a larger uncertainty set or for tighter specifications, when no robust solution exists. The challenge is how to marry robust and adaptive control to get the best of both.

6. Adaptive control from a robust perspective

Figure 6 shows the *trade-off* in all feedback design: If the plant uncertainty increases, more feedback gain is needed to maintain the same specifications. If the specifications get tighter more feedback gain is needed if the plant uncertainty remains unchanged. The trade-off will be illustrated with a scalar example.

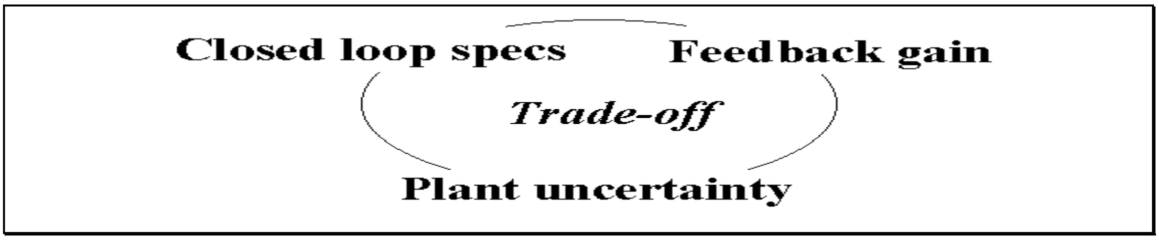


Figure 6: The trade-off in feedback design.

Referring to Figure 1, let $\boldsymbol{P}(\boldsymbol{s}) = \boldsymbol{k} \in [\boldsymbol{k}_{\min}, \boldsymbol{k}_{\max}]$ be a scalar gain plant, and $\boldsymbol{G}(\boldsymbol{s}) = \boldsymbol{g} > 0$ a scalar compensator. Let the plant uncertainty be defined by $\boldsymbol{k}_{\max}/\boldsymbol{k}_{\min}$. Referring to (2), let the specification for the closed loop uncertainty Δ be given by

$$\Delta = \frac{\max_{\boldsymbol{k}} |\bar{\boldsymbol{S}}|}{\min_{\boldsymbol{k}} |\bar{\boldsymbol{S}}|} = \frac{\boldsymbol{k}_{\max}}{\boldsymbol{k}_{\min}} \cdot \frac{1 + \boldsymbol{k}_{\min}\boldsymbol{g}}{1 + \boldsymbol{k}_{\max}\boldsymbol{g}} \le \frac{\boldsymbol{b}}{\boldsymbol{a}} \tag{6}$$

where $\boldsymbol{b} > \boldsymbol{a}$ are given, and $\bar{\boldsymbol{S}} = \boldsymbol{PG}/(1 + \boldsymbol{PG})$ is the complementary sensitivity function. A plot of Δ as a function of $\boldsymbol{g}$ is shown in Figure 7, for two different values of $\boldsymbol{k}_{\max}/\boldsymbol{k}_{\min}$. Clearly the trade-off mentioned above holds.

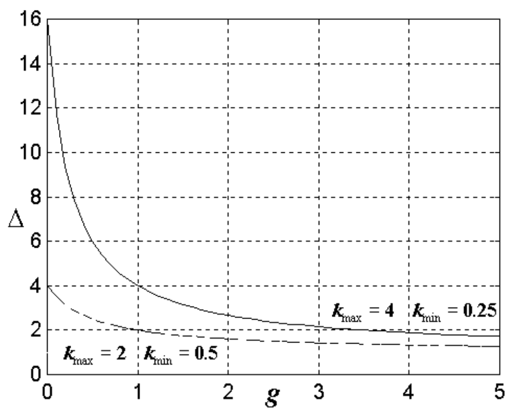


Figure 7: The remaining closed loop uncertainty Δ as a function of the feedback gain $\boldsymbol{g}$ for two different values of the open loop $\boldsymbol{k}_{\max}/\boldsymbol{k}_{\min}$.

With QFT, the *trade-off* is apparent at each frequency. This is illustrated with the following example. Consider the uncertain plant

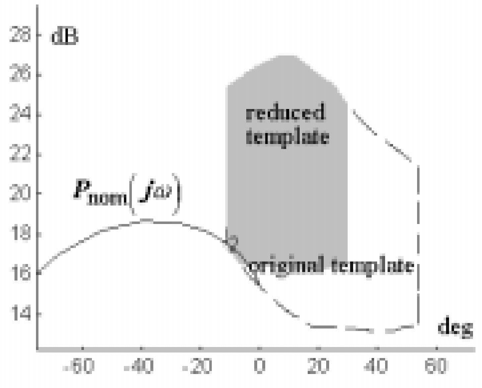
$$P(s) = k\frac{s+a}{1+2\zeta s/\omega_n + s^2/\omega_n^2}e^{-\tau s},\; k\in[2,5],\quad a\in[1,3],\quad \zeta\in[0.3,0.6],\; \omega_n\in[4,8],\quad \tau\in[0,0.05] \quad (7)$$

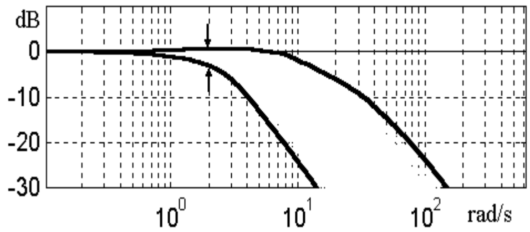
The uncertainty at each frequency, ω, is defined as the value set (Sánchez-Peña and Sznaier, 1998) , or *template* (Horowitz, 1963), $\{P(j\omega)\}$. In Figure 8 (upper left), $\{P(2j)\}$ is illustrated. Referring to (2), assume that the original servo specification in Figure 8 (upper right) has to be satisfied. From the figure it transpires, that the remaining closed loop gain uncertainty at 2 rad/s must satisfy

$$\Delta(2) = \frac{\max_k\left|\bar{S}(2)\right|}{\min_k\left|\bar{S}(2)\right|} \le \text{ 3.73 dB} \quad (8)$$

where $\bar{S}(j\omega)$ is the complementary sensitivity function. The resulting Horowitz bound, $\partial B_L(2j)$, is shown in Figure 8 (lower left). Notice that the gain of the bound depends on the phase. Assume now that a tighter specification is desired for which $\Delta(2) \le$2.13 dB. Then it follows that the new Horowitz bound for 2 rad/s is about 5 dB higher than before, see Figure 8, implying that the feedback controller gain must increase by about 5 dB. If, however, at the current operating condition e.g. the plant zero is less uncertain, such that $a\in[2.5,3]$, and this can be detected by an on-line parameter estimation algorithm, then the templates and Horowitz bounds can be recomputed. In Figure 8 the reduced uncertainty template for 2 rad/s is displayed. We notice that the template is considerably thinner than the original fat one. Therefore, the ensuing Horowitz bound for the tighter specification will have lower gain (Figure 8) and almost become equal to the original Horowitz bound valid for the original plant and the original uncertainty. Hence the trade-off in Figure 6 is apparent: A tighter specification can be off-set by increased plant knowledge.

But why not increase the feedback gain when larger plant uncertainty or tighter specifications require it? This can of course be done to a certain extent, and that is the basis of robust control. There are however fundamental limitations in feedback control (Seron et al., 1997). Any real-life plant includes delay or is





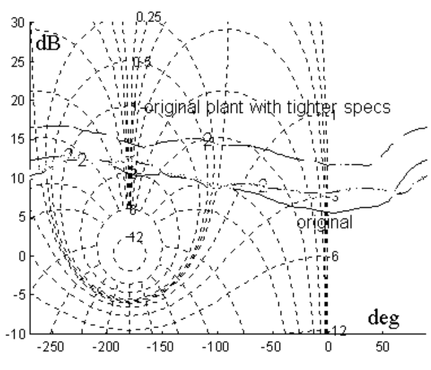


Figure 8:

Upper left: Original template, $\{P(2j)\}$, from (7); the nominal plant with k=2, a=3, ζ =0.6, ω_n =4, τ =0.05; and the reduced template which differs from (7) in that $a\in[2.5,3]$.

Upper right: The original closed loop servo specification. $\Delta(2)$ in (8) is the difference between the two arrows.

Lower left: The Horowitz bounds $\partial B_L(2j)$ for the original template and the original specification ("original"); the original template and tighter specifications ("...tighter specs"); and the reduced template with tighter specifications (unmarked).

non minimum-phase, or has large phase uncertainty at high frequencies. Then the phase margin requirement together with Bode's gain-phase relationship imposes a bandwidth limitation, and hence a limit on the allowed feedback gain. Moreover, the sensor noise is amplified at the plant input by $-G/(1+PG)$, see Figure 1. High feedback compensator gain is not wanted at the sensor noise frequencies, since it may cause actuator saturation and wear or require the use of a more expensive, low noise sensor.

By decreasing plant uncertainty, adaptation fights the fundamental feedback gain limitation, and shifts the *trade-off* in favor of tighter closed loops specifications. See Figure 9.

The landmark paper Yaniv *et al*. (1990) presents an algorithm how to adapt the parameters of a robust controller when more plant knowledge becomes available, and demonstrates the benefits. Gain adaptation of a robust controller is described in Gutman *et al*. (1988) and Zhou and Kimura (1994).

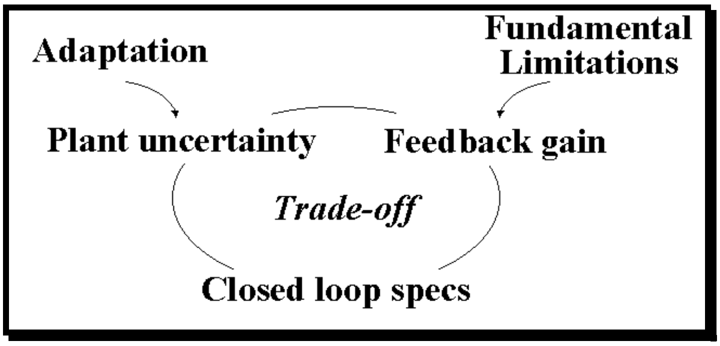


Figure 9: The trade-off in feedback design with adapatation.

7. The rôle of adaptation

The rôle of adaptation is to identify the plant templates $\{P_i(j\omega)\}$ at those frequencies that constrain the design, i.e. in QFT parlance, the frequencies at which $L_{\mathrm{nom}}(j\omega) \in \partial B_L(j\omega)$. The identification should be *helpful* for the design purpose to satisfy the specifications with least possible feedback gain, and could therefore be selective with respect to what plant parameters to estimate. In Figure 8 the identification of one parameter only was sufficient to decrease the template size so that the feedback gain requirement decreased by 5 dB. Figure 10 illustrates the case when there is a sensitivity specification, $|S| \leq 6$ dB that is satisfied tightly for some frequency. The identification of a smaller template (shaded area) is helpful only if it increases the distance from the template to the 6 dB sensitivity locus. Moreover, the adaptive controller should be able to redesign or retune the robust controller on which it is based, and switch the controller from one robust controller to another, in order to keep the closed loop system robust and quasi-LTI.

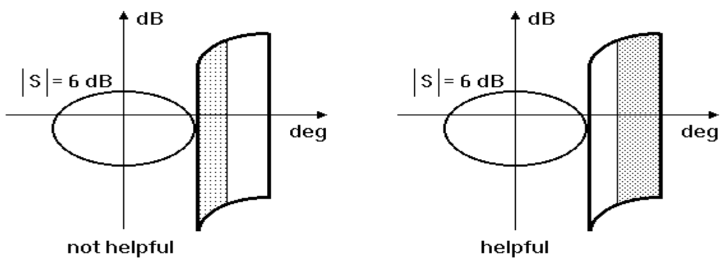


Figure 10: Helpful and unhelpful identification of a smaller template.

8. Adaptive robust control

It is however not sufficient to base the redesign or re-tuning of the robust controller on only the helpfully identified templates. Certainly equivalence is not suitable. The currently applied robust controller must be based on templates that incorporate plant cases to which the plant may drift or jump during the period to the next controller update. The identification may be based on probing at selected frequencies. We call this paradigm Adaptive Robust Control. A block diagram is found in Figure 11.

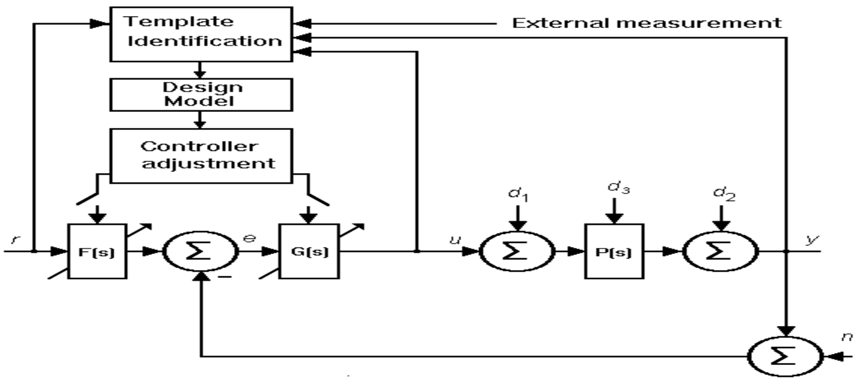


Figure 11. The block diagram for Adaptive Robust Control.

Composite templates can e.g be constructed as follows, see Figure 12. Before taking into account template extension due to possible plant drift and plant jumps, the identified template may even be a frequency function point estimate, e.g. from some common recursive estimation algorithm (Åström and Wittenmark,1995). A probabilistic template, e.g. the point estimate and its 1σ ellipse (in the Nyquist diagram) could constitute the template on which a high performance design is based, e.g. satisfying a servo specification (2). A worst case template, e.g. identified with set membership methods (Gutman, 1994, Rotstein et al, 1998) could serve as the template for stability design only. Finally, the trade-off between specifications, design template size, speed of plant drift and size of plant jumps, and the speed of on-line identification and controller updating can be illustrated as in Figure 13.

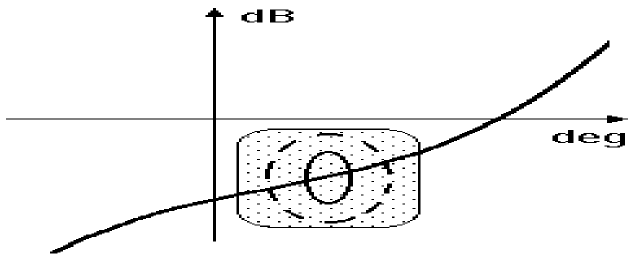


Figure 12: The identified template (solid), the probabilistic template for performance design (dashed), and the worst case template for stability design only (shaded), before template extension due to possible plant drift and plant jumps.

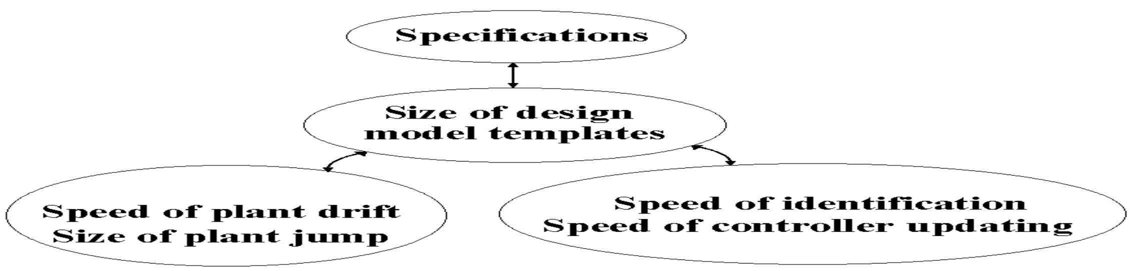


Figure 13: The Trade-Off in Adaptive Robust Control. A long identification time may give accurately identified templates, but will still require large design templates due to the possible plant drift and jumps.

9. Conclusions

A lot of research remains to be done before Adaptive Robust Control is in place whereby Adaptive Control solidly lies on Robust Control. Current on-line identification routines have to be developed further to identify templates in the required, selective, way. One useful idea may be found in Galperin *et al*. (1996). Few algorithms for switched adaptation of robust controllers have been published, Yaniv *et al*. (1990) being a landmark exception. Nevertheless one should mention Nordin (2000) as a very successful application of the ideas presented in this paper.

So, what about fidelity or a free relationship in the marriage between Robust and Adaptive Control? Obviously, Adaptive Control should stick to fidelity. Robust Control may however fiddle around, but not with too fat templates.

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