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Nonlinear controller tuning based on a sequence of identifications of linearized time-varying models

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ABSTRACT

A novel algorithm for tuning controllers for nonlinear plants is presented. The algorithm iteratively minimizes a criterion of the control performance. In each iteration one experiment is performed with a reference signal slightly different from the previous reference signal. The input–output signals of the plant are used to identify a linear time-varying model of the plant which is then used to calculate an update of the controller parameters. The algorithm requires an initial feedback controller that stabilizes the closed loop for the desired reference signal and in its vicinity, and that the closed-loop outputs are similar for the previous and current reference signals. The tuning algorithm is successfully tested on a laboratory set-up of the Furuta pendulum.

1. Introduction

Most methods for controller tuning rely on a mathematical model of the plant. The model is necessary to calculate the derivative of the output signal with respect to the controller parameters. See, e.g., Trulsson and Ljung (1985) on linear systems and, e.g., Harris (1994) on nonlinear systems. In this contribution a novel approach is presented: instead of identifying a, more or less, global nonlinear model of the plant, a linear time-varying model is identified along a trajectory of the plant. The time-varying model is then used to compute the derivative of the plant output with respect to the controller parameters. Since the model is valid only for a particular trajectory, the identification step has to be repeated in each iteration, i.e., after each change of the controller parameters.

The algorithm requires an initial, parameterized, feedback controller that stabilizes the closed loop for the desired reference signal and in its vicinity. This is illustrated in Fig. 1. It is also assumed that the plant can be described locally by a linear model which may be slowly time-varying along the considered neighboring trajectories. The proposed algorithm is then used to tune the controller parameters so that a control goal is optimized.

The algorithm is described by the following steps.

Algorithm 1.1. *Basic steps for tuning the controller.*

1. *Perform one experiment on the plant with a desired reference signal.*
2. *Identify a linear time-varying plant model describing the input–output data.*
3. *Use the model to compute the derivative (with respect to the controller parameters) of the criterion of the control objective along the trajectory.*
4. *Update the controller parameters.*
5. *Repeat from step 1 if the control objective is not satisfied.*

The user has to choose a linear model structure for the plant and an identification method with which to identify its time-varying parameters. It is then assumed that the reference signal and the sequence of updated feedback controllers are such that the plant input signal becomes persistently exciting in order to ensure identifiability of the plant model parameters.

The linearization of the plant must change slowly during an experiment, otherwise it is not possible to obtain a good linear time-varying model. This can be assured if the reference signal is changing slowly between different operating regimes, i.e., if it is dominated by low frequencies. However, there must be some high frequency contents of low amplitude, otherwise the dynamics cannot be identified.

It should be noted that the identification of the linear time-varying model is done off-line. This implies that, in contrast to

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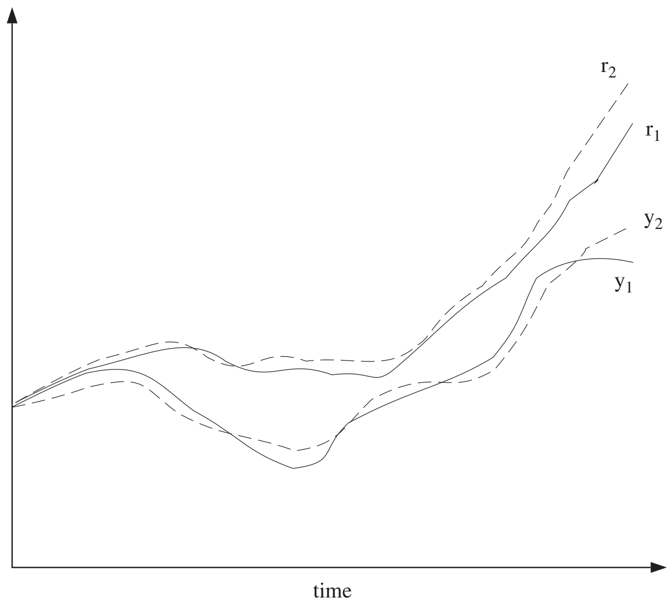

Fig. 1. Comparison of reference signals and output signals of two experiments:the existing controller is assumed to stabilize the system along the trajectory in such a way that similar reference signals give output signals close to one another.

normal recursive identification, not only preceding but also successive data are used to obtain a model at time *t*. Hence, the model quality and the tuning is improved.

The optimization is done with respect to a desired reference signal and the obtained controller is then optimal only for this specific reference signal. Above it was assumed that the initial stabilizing controller caused the closed-loop outputs to be similar for reference signals in the neighborhood of the desired reference signal. If this property is retained for the optimized controller then this controller will be nearly optimal also for reference signals in the mentioned neighborhood.

The problems for which the results of this paper may be useful include the following:

- In batch processes, e.g., in the chemical industry, it is common that a controlled variable, e.g., temperature, has to track a given reference trajectory. When a model of the process is unavailable or only known with large uncertainty, but the process is equipped with a functioning feedback controller, the proposed procedure may be used to optimize the controller. In Sjöberg and Agarwal (1998) an example of this is given.
- A common form of adaptive control is the so-called *gain scheduling* (Aström & Wittenmark, 1995) whereby different controllers have been designed for a number of operating points. When the actual operating point is at or near a design operating point the corresponding controller output is input to the plant. The present procedure may be used to tune controllers optimized for a number of reference trajectories, thus constructing a trajectory-dependent gain scheduling. An example of gain-scheduling controller tuning is given in Section 7. It should, however, be pointed out that gain scheduling may be non-trivial for, e.g., multiple model systems or linear time-varying systems, see, e.g., Murray-Smith and Johansen (1997) and Johansen and Foss (1999)and other articles in that special issue, and Leith and Leithead (2000).
- The proposed method can also be used to tune linear controllers in connection with nonlinear plants. For example, the commonly used PI- and PID can be tuned in this way. The second example in Section 7 demonstrates PID-tuning for the Furuta pendulum (Aström & Furuta, 2000; Furuta, 2003; Furuta, Yamakita, & Kobayashi, 1991; Furuta, Yamakita, Kobayashi, & Nishimura, 1991).
- Under the assumption that a controller is known to globally stabilize the plant when the controller parameters belong to a set, one could use the procedure in this paper to tune the parameter values with respect to a number of reference trajectories that are weighted into one criterion. One optimized global controller is then obtained.

The current method, a preliminary version of which was presented as Sjöberg and Agarwal (1997), can be compared with many contributions in the control literature. Since the model is valid only locally around a trajectory, the new algorithm can be seen as a special case of *modeling for control*. However, most other works on modeling for control address the frequency contents of the input signal for linear systems, see, e.g., Gunnarsson and Hjalmarsson (1994), Aström and Nilsson (1994) and Gevers (1993). The algorithm presented in this paper can be seen as a novel type of identification for control where the time-varying identification is a way to take nonlinearities of the plant into account.

Iterative feedback tuning is suggested for linear systems in Hjalmarsson, Gevers, Gunnarsson, and Lequin (1998) and Hjalmarsson, Gunnarsson, and Gevers (1995), where instead of using a model, three experiments are performed in each iteration. For nonlinear plants that method requires in each iteration a number of experiments equal $n+1$ or $n+2$, where n is the number of controller parameters, see Sjöberg and Agarwal (1996), De Bruyne, Anderson, Gevers, and Linard (1997)or Sjöberg et al. (2003). If the controller contains many parameters this may be too many experiments to be applicable. The model-based tuning algorithm presented here is a way to reduce the number of experiments in each iteration to one. The proposed method has several user aspects in common with iterative feedback tuning. For example, none of the methods give any guarantees for closed-loop stability. Instead, if stability is a problem, then this has to be assured using some additional method. See Hjalmarsson et al. (1998) for a discussion on such issues.

The present contribution can be seen as an extension of Sjöberg et al. (2003). The difference is that here one experiment per controller update iteration is needed to identify a time-varying linear plant model in order to compute the derivatives of the output with respect to the controller parameters, while the method in Sjöberg et al. (2003) is model free, but, as mentioned above, requires $n+1$ or $n+2$ experiments per iteration to compute the derivatives. Otherwise the two methods are identical. Common issues and assumptions associated with the iterative combination of identification, derivative estimation, and control design within a noisy and nonlinear dynamic context are mostly treated in Sjöberg et al. (2003).

There is also a close connection to *extremum control* (Aström & Wittenmark, 1995; Sternby, 1980; Wittenmark & Urquhart, 1995) which is a type of adaptive control where an on-line search in the controller parameter space is conducted in order to minimize a criterion. If the method is applied to tune the controller parameters on-line it becomes an extremum controller. The advantage of the new algorithm is that it does not require the controller parameters to be moved away from the optimum in order to compute the derivative of the criterion with respect to the controller parameters which is the case with conventional extremum control.

Finally, there is a connection to *quantitative feedback theory* for smooth nonlinear systems, see Horowitz (1982, 1992), in which linear input–output transfer function models are identified for different desired outputs of the uncertain nonlinear plant. Based

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on these models, one linear controller is designed which is robust both with respect to performance and stability, using a proof based on Schauder's fixed point theorem. As suggested in point three above, the method could be used to design and optimize a robust controller.

The novel idea in this paper is hence to use a linear time-varying description instead of a global nonlinear model, or no model at all, to compute the derivatives of the output with respect to the controller parameters. The great advantage lies in the fact that linear models are easier to specify whereas global nonlinear descriptions require considerable insight into the plant behavior, and numerous experiments. However, if a good global nonlinear model can be obtained it is probably better to use parametric optimization methods as described in, e.g., Harris (1994) and Monje, Vinagre, Feliu, and Chen (2008). Such a model can, of course, also be expanded as a linear time-varying model along a trajectory, see, e.g., Hu, Kumamaru, and Inoue (1996).

The paper is organized in the following way. In Section 2 the problem formulation and the control objective are stated. In Section 3 a time-varying description of a nonlinear plant is given. How to obtain the derivative of the output with respect to the control parameters is described in Section 4, and different possibilities for parameterizing the linear time-varying model are presented in Section 5. Advantages and limitations of the proposed scheme are discussed in Section 6. Two application examples are given in Section 7, including laboratory experiments with the Furuta pendulum. The paper is concluded in Section 8.

2. Control objective and problem formulation

Given a nonlinear plant, assume there exists a controller of the following form:

$$u(t) = g(\rho, y(t), y(t-1), \ldots, y(t-n_y), u(t-1), \ldots, u(t-n_{u_1}), r(t), \ldots, r(t-n_r)), \tag{1}$$

which stabilizes the plant for some initial controller parameter vector value $\rho = \rho_0$. The control signal $u(t)$ depends on present and past plant outputs y and reference signals r. The initial controller with $\rho = \rho_0$ as well as the updated vectors ρ_i, $i = 1, 2, \ldots$ are assumed to stabilize the system along the trajectory of an experiment. The goal is then to tune ρ so that the performance of the controller is optimized with respect to some criterion.

By the Internal Model Principle, as expressed, e.g., in Morari and Zafiriou (1989), it follows from the assumption that the closed-loop system is stabilized by the initial controller, that the plant can be described by

$$\begin{aligned} z(t) &= f(z(t-1), \ldots, z(t-n_z), u(t-1), \ldots, u(t-n_{u_2})), \\ y(t) &= z(t) + v(t), \end{aligned} \tag{2}$$

where $f(\cdot)$ is an unknown nonlinear function and $v(t)$ is a zero-mean, not necessarily white, noise signal. This description also follows from an extension of the Takens embedding theorems (Stark, 1999; Stark, Broomhead, Davies, & Huke, 1997). Compare also with Horowitz (1992, 309pp). It will also be assumed that $f(\cdot)$ is a smooth function. The integers n_z and n_{u_2} are unknown, but they are related to the number of known shifts in the stabilizing controller, n_r and n_y. By the Internal Model Principle, one may, e.g., let $n_z = n_y$ and $n_{u_2} = n_r$.

2.1. The control criterion and its minimization

Let the desired plant output be described as a filtered version of the reference signal

$$y_d(t) = T_d\, r(t).$$

The filter T_d is typically linear time-invariant, but it could also be nonlinear or time-varying.

A natural design objective is to minimize some norm of the deviation of the plant output $y(t)$ from the desired output $y_d(t)$. Indicating this norm by $J(\rho)$ the desired value of the controller parameters becomes

$$\rho^* = \arg\min_{\rho} J(\rho). \tag{3}$$

The criterion J could be any measure of the fit. In this contribution, however, the attention is restricted to quadratic measures of the form

$$J(\rho) = \sum_t \left[\frac{1}{2}\, \mathrm{E}[(L_y(y(\rho) - y_d))^2] + \frac{\gamma}{2}\, \mathrm{E}[(L_u u(\rho))^2] \right], \tag{4}$$

where the expectations are with respect to the noise $v(t)$ in (2). The design parameter γ is used to monitor the trade-off between performance and control effort. The filters L_y and L_u can be used to emphasize certain frequencies. They will be set to unity in the sequel since they do not change the general derivation of the algorithm.

To minimize (4), the following equation must be solved with respect to ρ:

$$0 = J'(\rho) = \mathrm{E}[(y(\rho) - y_d) y'(\rho)] + \gamma\, \mathrm{E}[u(\rho) u'(\rho)] \tag{5}$$

which is done by a numerical search

$$\rho_{i+1} = \rho_i - \mu_i R_i^{-1} J'(\rho_i). \tag{6}$$

In each iteration i the parameters are changed in a descent direction. The matrix R_i is typically an approximation of the Hessian matrix which changes the search from the steepest-descent to a more favorable direction (Dennis & Schnabel, 1983). The step length μ_i should be chosen so that a down-hill step is obtained. Conditions for convergence to a local minimum are briefly discussed in Sjöberg et al. (2003). A discussion about the rate of convergence is outside the scope of this paper, instead see, e.g., Dennis and Schnabel (1983).

The tricky part in (5) and (6) is the computation of the derivatives $y'(\rho)$ of the output and $u'(\rho)$ of the control signal. The contribution of this paper is to obtain an estimate of the derivatives by linearized time-varying modeling of the plant. Notice that since the model is valid only along a particular trajectory, which in turn depends on ρ, the modeling has to be repeated in each iteration of (6).

3. Linearized time-varying description of the plant

In this section the linearized time-varying description of the plant is given. This description is then used in the following section to obtain an expression for the derivative of the plant output and the control action with respect to the controller parameters.

The changes of the trajectories of an experiment due to a small perturbation of the controller parameters are described with Taylor expansions. The expressions for the derivatives are then obtained by letting the parameter perturbation go to zero. It will be shown that when the algorithm is used, no experimental change of the parameter vector is necessary in order to obtain the derivatives.

Given a reference signal $\{r_i(t)\}_{t=1}^N$ consisting of N time samples, an experiment labeled i can be performed on the plant with the current controller parameters ρ_i. This gives trajectories of the plant output $\{y_i(t)\}_{t=1}^N$ and of the control signal $\{u_i(t)\}_{t=1}^N$. Introducing the following variables as differences from $\{y_i(t), u_i(t), r_i(t)\}$:

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$$\begin{aligned}
\Delta y(t) &= y(t) - y_i(t),\\
\Delta u(t) &= u(t) - u_i(t),\\
\Delta r(t) &= r(t) - r_i(t),\\
\Delta z(t) &= z(t) - z_i(t),\\
\Delta \rho(t) &= \rho(t) - \rho_i(t).
\end{aligned} \tag{7}$$

The output (2) can be described as

$$y(t) = z_i(t) + \Delta z(t) + v(t). \tag{8}$$

The plant (2) is assumed to be smooth. Hence, if $\{\Delta z(t-n)\}_{n=1}^{n_z}$ and $\{\Delta u(t-n)\}_{n=1}^{n_{u_2}}$ are small then (2) can be expressed by a Taylor expansion around $z_i(t)$ and $u_i(t)$

$$\begin{aligned}
z(t) = z_i(t) &+ \sum_{n=1}^{n_z} \frac{\partial f(\cdot)}{\partial z(t-n)} \Delta z(t-n)\\
&+ \sum_{n=1}^{n_{u_2}} \frac{\partial f(\cdot)}{\partial u(t-n)} \Delta u(t-n) + O(\varDelta_{\max}^2),
\end{aligned} \tag{9}$$

where the partial derivatives are evaluated at $\{z_i(t-n)\}_{n=1}^{n_z}$ and $\{u_i(t-n)\}_{n=1}^{n_{u_2}}$. The higher order terms are described by $O(\varDelta_{\max}^2)$, where $\varDelta_{\max}$ indicates the largest $\varDelta$ in the equation, and $O(x) \to 0$ when $x \to 0$. This expression can be found for each t of the experiment, and an expansion of the function $f(\cdot)$ along the trajectory $\{z_i(t), u_i(t)\}_{t=1}^{N}$ is obtained. Of course, since $\{z_i(t)\}_{t=1}^{N}$ is not available, and the system $f(\cdot)$ is unknown, the expansion is only a formal description. Instead, the idea is to identify the partial derivatives in (9) by using experimental data. This is discussed in the next section.

In the same way as the plant was linearized, the controller $u(t)$ in (1) can also be expressed by a Taylor series around $\{y_i(t), u_i(t), r_i(t)\}_{t=1}^{N}$ and ρ_i

$$\begin{aligned}
u(t) = u_i(t) &+ \sum_{n=0}^{n_y} \frac{\partial g(\cdot)}{\partial y(t-n)} \Delta y(t-n) + \sum_{n=1}^{n_{u_1}} \frac{\partial g(\cdot)}{\partial u(t-n)} \Delta u(t-n)\\
&+ \sum_{n=0}^{n_r} \frac{\partial g(\cdot)}{\partial r(t-n)} \Delta r(t-n) + g(\{y_i, u_i, r_i\})' \, \Delta\rho + O(\varDelta_{\max}^2),
\end{aligned} \tag{10}$$

where $g(\{y_i, u_i, r_i\})'$ is the partial derivative of the control signal with respect to the parameters ρ evaluated along the trajectory $\{y_i(t), u_i(t), r_i(t)\}_{t=1}^{N}$.

The relation between $\Delta r(t)$, $\Delta u(t)$, and $\Delta y(t)$ can now be described as a time-varying system. To see this the following notation is introduced:

$$\begin{aligned}
F &= F(t, q^{-1}) \doteq \frac{1}{A} B,\\
A &\doteq 1 - f_{z\,1}(t) q^{-1} - \cdots - f_{z\,n_z}(t) q^{-n_z},\\
B &\doteq f_{u\,1}(t) q^{-1} + \cdots + f_{u\,n_{u_2}}(t) q^{-n_{u_2}},\\
R &\doteq 1 - g_{u\,1}(t) q^{-1} - \cdots - g_{u\,n_{u_1}}(t) q^{-n_{u_1}},\\
S &\doteq -g_{y\,0}(t) - g_{y\,1}(t) q^{-1} - \cdots - g_{y\,n_y}(t) q^{-n_y},\\
T &\doteq g_{r\,0}(t) + g_{r\,1}(t) q^{-1} + \cdots + g_{r\,n_r}(t) q^{-n_r},
\end{aligned} \tag{11}$$

where q is the shift operator, i.e., $q^{-1}u(t) = u(t-1)$, and the partial derivatives, evaluated along the trajectories $\{z_i(t), u_i(t)\}_{t=1}^{N}$ and $\{y_i(t), u_i(t), r_i(t)\}_{t=1}^{N}$, have been abbreviated in the following way:

$$\begin{aligned}
f_{z\,n}(t) &\doteq \left.\frac{\partial f(\cdot)}{\partial z(t-n)}\right|_{\{z_i(t-n)_{n=1}^{n_z}, u_i(t-n)_{n=1}^{n_{u_2}}\}},\\
f_{u\,n}(t) &\doteq \left.\frac{\partial f(\cdot)}{\partial u(t-n)}\right|_{\{z_i(t-n)_{n=1}^{n_z}, u_i(t-n)_{n=1}^{n_{u_2}}\}},\\
g_{y\,n}(t) &\doteq \left.\frac{\partial g(\cdot)}{\partial y(t-n)}\right|_{\{y_i(t-n)_{n=0}^{n_y}, u_i(t-n)_{n=1}^{n_{u_1}}, r_i(t-n)_{n=0}^{n_r}, \rho=\rho_i\}},\\
g_{u\,n}(t) &\doteq \left.\frac{\partial g(\cdot)}{\partial u(t-n)}\right|_{\{y_i(t-n)_{n=0}^{n_y}, u_i(t-n)_{n=1}^{n_{u_1}}, r_i(t-n)_{n=0}^{n_r}, \rho=\rho_i\}},\\
g_{r\,n}(t) &\doteq \left.\frac{\partial g(\cdot)}{\partial r(t-n)}\right|_{\{y_i(t-n)_{n=0}^{n_y}, u_i(t-n)_{n=1}^{n_{u_1}}, r_i(t-n)_{n=0}^{n_r}, \rho=\rho_i\}}.
\end{aligned} \tag{12}$$

Note that F, A, and B are unknown since f is unknown, while R, S, and T may be computed from the known g. The new notation and the Taylor expansions (9) and (10) give the linearized time-varying system

$$\begin{aligned}
\Delta z(t) &= F \Delta u(t) + O(\varDelta_{\max}^2),\\
\Delta y(t) &= \Delta z(t) + v(t) + O(\varDelta_{\max}^2)
\end{aligned} \tag{13}$$

and the linearized time-varying controller

$$\Delta u(t) = \frac{1}{R}\left(g(\{y_i, u_i, r_i\})' \Delta\rho - S\Delta y(t) + T\Delta r(t)\right) + O(\varDelta_{\max}^2). \tag{14}$$

Plugging in the control signal (14) in the plant equation (13) gives us the linearized closed-loop system

$$\Delta y(t) = F\frac{1}{R}\left(g(\{y_i, u_i, r_i\})' \Delta\rho - S\Delta y(t) + T\Delta r(t)\right) + v(t) + O(\varDelta_{\max}^2). \tag{15}$$

This relation will be used to obtain expressions for the derivatives of the output and the control signal.

4. The output and input derivatives

In this section it is demonstrated how the output and input derivatives with respect to the controller parameters can be obtained using the linearized system description from the previous section.

Once the trajectories $\{y_i(t), u_i(t), r_i(t)\}_{t=1}^{N}$ and $\{z_i(t), u_i(t)\}_{t=1}^{N}$ have been obtained in an experiment, they are used merely as a series of points around which the Taylor expansions (9) and (10) are computed. This means that $y_i(t)$ in (7) does not depend on ρ and, hence, the derivative of the linearized plant becomes identical with the derivative of the original plant (2), i.e.,

$$y'(t) \equiv \frac{\mathrm{d}y}{\mathrm{d}\rho} = \frac{\mathrm{d}\Delta y}{\mathrm{d}\Delta\rho} \equiv \Delta y'(t).$$

Using this relation the derivative with respect to controller parameters can be obtained using the linear time-varying plant. Taking the derivative of (15) with respect to $\Delta\rho$ gives

$$\Delta y'(t) = F\frac{1}{R}\left(g(\{y_i, u_i, r_i\})' - S\Delta y'(t)\right) + O(\varDelta_{\max}). \tag{16}$$

This gives us the following expression for the derivative:

$$y'(t) = \frac{1}{1 + F\frac{1}{R}S} F \frac{1}{R} g(\{y_i, u_i, r_i\})' + O(\varDelta_{\max}). \tag{17}$$

If the control-signal penalty γ is non-zero in the control criterion (4), then the derivative of the control signal is also needed and it can be obtained from (14)

$$u'(t) = \Delta u'(t) = \frac{1}{R} g(\{y_i, u_i, r_i\})' - \frac{1}{R} S y'(t) + O(\varDelta_{\max}). \tag{18}$$

This expression also follows directly from the original expression of the controller (1).

Since $\Delta_{\max}$ may be chosen arbitrary small, the error terms in (17) and (18) can be made arbitrary small. Hence, the exact expressions of the derivatives of the output and the control signal with respect to the controller parameters are obtained from (17) and (18) by setting the error terms to zero.

The only practical problem which prevents us from using (17) and (18) directly is that using the time-varying plant description, F, is not known. In the following section possibilities for estimating it are discussed.

5. The linear time-varying model

After having derived expressions (17) and (18) for the output and control derivatives it remains to specify how the linear time-varying filter $(1/(1+F(1/R)S))F(1/R)$ should be obtained in each iteration of the tuning algorithm. This is a design step and there are many different approaches. The best choice depends on the problem at hand, and a rather general discussion about the different possibilities will be given. One possible design approach will then be discussed in more detail.

A general requirement for the tuning method to be applicable is that it is possible to obtain a good time-varying model. This is possible if the the time-varying description of the plant (13) changes slowly with time, i.e., if there is a small change of the linear description between consecutive time samples. This can be described by considering the frequency contents of the reference signal.

- Assume that the reference signal is dominated by low frequencies superimposed with a high frequency part of smaller amplitude. Then the linearization of the system depends on the low frequency component which describes a changing operating point of the system. The high frequency part of the reference signal describes deviations around the low frequency part, and if the amplitudes of these deviations are small, then the they will only influence the linearization marginally.

Using this frequency description of the reference signal, it follows that the high frequency component is necessary to obtain excitation of the dynamic of the system. Refer to Ljung (1999) for a discussion on the necessity and choice of "persistently exciting" excitation signals.

Consider now the time-varying description (17). In principle, only the unknown F in (17) needs to be estimated, since R and S can be computed by definition (12). However, it is also possible to estimate the closed-loop system $(1/(1+F(1/R)S))F(1/R)$ directly, as shown in the simulation example in Section 7. These two possibilities merely represent different ways of closed-loop identification; general aspects on closed-loop identification are discussed in, e.g., den Hof and Schrama (1995) and Fogel and Huang (1982).

The expression for the time-varying system (13) can be re-written as

$$\begin{aligned} y(t) &= y_i(t) + \Delta y(t) = y_i(t) + F(\Delta u(t) + u_i(t) - u_i(t)) + v(t) + O(\Delta_{\max}^2) \\ &= y_i(t) + Fu(t) - Fu_i(t) + v(t) + O(\Delta_{\max}^2) \\ &= Fu(t) + v(t) + b_0(t) + O(\Delta_{\max}^2), \end{aligned} \tag{19}$$

where $b_0(t) = y_i(t) - Fu_i(t)$ is a time-varying parameter describing the DC-level of the output signal, i.e., it describes the influence of the low frequency part of the reference signal. The derivations above leading to (19) can easily be extended to include the case of filtered noise rather than the assumed output noise. Therefore relationship (19) can be modeled by a general linear time-varying model given by

$$y(t) = G(t, q^{-1})u(t) + H(t, q^{-1})e(t) + b_0(t), \tag{20}$$

where $G(t, q^{-1})$ is the plant dynamics and $H(t, q^{-1})$ is the noise dynamics at time t. The DC-level parameter $b_0(t)$ corresponds to the operating point around which the linearization is done at time t. Hence, $b_0(t)$ is a special type of parameter and in some situations it should be handled differently from the other parameters. It is, for example, possible that $b_0(t)$ changes fast at certain time instances as the response to fast changes in the reference signal. Then a possible solution is to perform two experiments with similar reference signals and to form $\delta r = r_1(t) - r_2(t)$, $\delta y = y_1(t) - y_2(t)$ etc., and to use the δ data sequences in the identification. Then the parameter $b_0(t)$ does not have to be included in the model, since its contribution is canceled in the subtraction of the signals from the two experiments, i.e., (20) can be replaced by

$$\delta y(t) = G(t, q^{-1})\delta u(t) + H(t, q^{-1})e(t). \tag{21}$$

In some situations there might be prior knowledge available to guide the choice of $G(t, q^{-1})$ and $H(t, q^{-1})$. Otherwise one can choose a black-box model structure such as ARX, OE, ARMAX, or BJ. See, e.g., Ljung (1999).

Here the general discussion is concentrated on the time-varying ARX model

$$y(t) = \frac{1}{A}Bu(t) + \frac{1}{A}e(t) + b_0(t) \tag{22}$$

with

$$A = 1 + a_1(t)q^{-1} + \cdots + a_{n_a}(t)q^{-n_a},$$

$$B = b_{n_k}(t)q^{-n_k} + \cdots + b_{n_k+n_b-1}q^{-n_k-n_b+1},$$

where n_a, n_b, and n_k specify the model orders and time delay, respectively. This can also be described as a time-varying linear regression

$$y(t) = \theta^{\mathrm{T}}(t)\varphi(t) \tag{23}$$

with

$$\theta(t) = [a_1(t), \ldots, a_{n_a}(t),\ b_{n_k}(t), \ldots, b_{n_k+n_b-1}(t),\ b_0(t)]^{\mathrm{T}}$$

and

$$\varphi(t) = [-y(t-1), \ldots, -y(t-n_a),\ u(t-n_k), \ldots, u(t-n_k-n_b+1),\ 1]^{\mathrm{T}}.$$

Note the 1 in the last position of the regressor. It has been included so that $b_0(t)$ can be estimated together with the other parameters.

The typical way to estimate time-varying models with noisy measurements is to use a recursive estimation algorithm like the Kalman filter or recursive least squares with a forgetting factor. See, e.g., Ljung and Soderstrom (1983) and Anderson and Moore (1979). Issues related to the validity of the model, influence of measurement noise and choice of excitation signal are also addressed in, e.g., Ljung (1999) and Sjöberg et al. (2003). Here there is an off-line situation. All data are available for identification, not only those for time samples up to t. Hence, there is no need to use a recursive algorithm just because the system description (20) is time-varying. Instead the parameters in (23) can be described as basis function expansions. For example, the parameter $a_1(t)$ can be expanded as

$$a_1(t) = \sum_{k=1}^{n} a_{1k}h_k(t), \tag{24}$$

where $\{h_k(t)\}_{k=1}^{n}$ are the basis functions and $\{a_{1k}\}_{k=1}^{n}$ are the new, time-invariant parameters. This is actually a re-parameterization of the model where the time-dependence is separated from the parameters. The basis expansion should be chosen so that the time dependent variation is largest where the changes in the plant dynamics are largest. For example, if it is known that the reference signal jumps from one level to another at some time instant t_0 and that the reference signal is relatively constant before and after this jump, then this motivates the following two basis functions:

$$h_1(t) = \begin{cases} 1, & t < t_0, \\ 0, & t \geqslant t_0, \end{cases} \quad h_2(t) = \begin{cases} 0, & t < t_0, \\ 1, & t \geqslant t_0. \end{cases}$$

On the other hand, a time-invariant model corresponds to one constant basis function, i.e., $n = 1$ and $h_1(t) = 1$ in (24).

Another approach, suggested in the example in Section 7, is to use a weighted criterion (Ljung, 1999), such that the observations around time t are more important than other observations for the estimation of the parameter values at t. The parameters at t are then obtained by minimizing the criterion

$$V(t,\theta) = \frac{1}{2N}\sum_{i=1}^{t} \lambda_t^{t-i}[y(i) - \theta^{\mathrm{T}}(i)\varphi(i)]^2 + \frac{1}{2N}\sum_{i=t+1}^{N} \lambda_t^{i-t}[y(i) - \theta^{\mathrm{T}}(i)\varphi(i)]^2. \tag{25}$$

Note that the first term of the right-hand side corresponds to the recursive least squares with forgetting factor (Ljung, 1999). The forgetting factor $\lambda_t \leqslant 1$ can be different for different t. This gives us the possibility to have more local models where the dynamics change fast and vice versa. It is also possible to choose shapes for the time-window other than the exponential one, e.g., a rectangular window could be possible.

The choice of λ_t in (25) gives a bias–variance trade-off for the time-varying model. With $\lambda_t = 1$ the model becomes time-invariant and the parameters are supported equally by all data. For $\lambda_t < 1$ the data close to t gives a larger contribution to the fit than those far away. A lower λ_t-value gives less bias and higher variance in the parameter estimate.

The choice of λ_t is also guided by the controller structure (1). The bias–variance trade-off in the modeling is just an intermediate step of the tuning of the controller parameters and it is the variance and bias of the controller parameters which are of primary interest. Hence, a small value of λ_t, giving a noisy model estimate, can be accepted if the time-varying model is used to estimate a controller with a low number of parameters.

Notice that this claim only holds if all the control parameters influence the controller for all, or at least many, of the time samples in the trajectory. For example, an adaptive pole placement controller, see, e.g., Aström and Wittenmark (1995) does not have this feature, the controller at a given t depends only on the current model at t. Hence, for an adaptive pole placement controller the error in the parameters at time t carry over directly to the control parameters.

On the other hand, if the controller is, in some sense, globally parameterized so that the controller parameters at a given t depend on the model at many different t values, then the influence of the model-parameter variance will decrease. The reason for this is that the controller parameters are a function of, loosely speaking, the mean value of several models at different t values. So for globally parameterized controllers one can use a lower value of λ_t since the variance contribution is averaged out. This gives a possibility to reduce the bias without increasing the variance in globally parameterized controllers.

6. Discussion

The traditional alternative to the proposed algorithm is to identify a global nonlinear model which can be used for the tuning. What are the advantages and disadvantages of the two methods? If a good global model is available, then it is probably a good idea to use it; if not, the proposed algorithm might be a useful alternative.

The main advantage with the proposed method is probably that the plant behavior is modeled only around the trajectory where it is needed. There are no parameters or data wasted on estimating the plant away from the trajectory. Choosing a nonlinear model structure is in many cases a very delicate matter. In some applications there is enough prior knowledge to construct a physical model that is parsimonious in the number of unknown parameters. However, in many applications one has to play around with nonlinear black-box models. Since there are many more possible choices for nonlinear black-box models than for linear ones it can be a time consuming and tedious task to find an appropriate structure—if at all possible. See, e.g., Sjöberg et al. (1995) and Cao, Rees, and Feng (1997a, 1997b) for aspects on this. Hence, the proposed method can be seen as a way to avoid the nonlinear identification problem. Instead of choosing an appropriate nonlinear model structure one has to choose among the family of linear time-invariant model structures (20). This is generally a much easier task. If criterion (25) is used, then in addition to the model structure, the forgetting factor λ_t has to be decided upon. However, as described in the previous section, if the controller is globally parameterized, the controller is expected to be robust with respect to the exact choice of λ_t.

A mixture of the two methods would be to update a global model in each iteration of (6). This corresponds to iterative identification for control in the linear case, described in, e.g., Gunnarsson and Hjalmarsson (1994).

It is, of course, possible to generalize the proposed algorithm and to include the results from several experiments in each iteration of the controller parameter update. Depending on the nature of the different trajectories this can give a more "global" controller spanning all the used trajectories. Such an approach would give a series of trajectories $\{r_k(t)\}$ along which the derivative $J'(\rho)$ of the control criterion is computed. The total criterion is then a weighted sum of the performances along the different trajectories. Note that if the unperturbed reference trajectory is chosen to be constant, $r_k(t) = r_k$, then a time-invariant linear model will be obtained.

7. Two examples

In Sjöberg and Agarwal (1998), a simulation example can be found where the algorithm is used to tune the controller of a chemical batch reactor. Here, another two examples are given. The first example illustrates, by simulation, the possible use of the algorithm to tune a gain-scheduling controller for a small nonlinear plant; the second example shows the tuning of a PID controller for a laboratory set-up of the Furuta pendulum.

7.1. Gain scheduling for a nonlinear plant

The plant is described by the following equation:

$$\begin{aligned} y(t) = {} & (1 - \sigma(t))(1.5\,y(t-1) - 0.56\,y(t-2)) \\ & + \sigma(t)(3\,y(t-1) - 2.26\,y(t-2)) + u(t-1) + v(t), \end{aligned} \tag{26}$$

where $v(t)$ is Gaussian white noise with standard deviation 0.05, and

$$\sigma(t) = \frac{1}{1 + \exp(-2\sqrt{y^2(t-1) + y^2(t-2)} + 8)}.$$

Note that the noise in (26) enters as process noise in violation with the measurement noise assumption in (2). This makes the

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illustration slightly more realistic, although the noise level is low. The desired closed-loop system is specified as

$$y_d(t) = \frac{0.3}{1 - 0.9q^{-1} + 0.2q^{-2}}\, r(t-1) \tag{27}$$

which is a low pass filtering of $r(t)$ with poles at 0.4 and 0.5.

For small values of $y(t)$ one has $\sigma(t) \approx 0$ and the plant is approximately linear,

$$y(t) \approx 1.5\,y(t-1) - 0.56\,y(t-2) + u(t-1) + v(t).$$

For large values of $y(t)$ it holds that $\sigma(t) \approx 1$ and the linear difference equation

$$y(t) \approx 3y(t-1) - 2.26\,y(t-2) + u(t-1) + v(t).$$

These linear approximations motivate control laws that place the dominant closed-loop poles at $\{0.4, 0.5\}$ according to (27). One gets

$$u_1(t) = -0.6y(t) + 0.36y(t-1) + 0.3r(t) \tag{28}$$

for small values of $y(t)$, and

$$u_2(t) = -2.1y(t) + 2.06y(t-1) + 0.3r(t) \tag{29}$$

for large values of $y(t)$.

A gain-scheduling controller (Aström & Wittenmark, 1995), $u_g(t)$ is then designed by combining the linear control laws $u_1(t)$ and $u_2(t)$ as

$$u_g(t) = (1-k)u_1(t) + ku_2(t), \tag{30}$$

where

$$k = \begin{cases} 0 & \text{if } y(t) \leqslant 1, \\ (y(t)-1)/3 & \text{if } 1 < y(t) < 4, \\ 1 & \text{if } y(t) \geqslant 4. \end{cases}$$

The desired output $y_d(t)$ is obtained according to (27) where $r(t)$ is a sum of a low- and a high-frequency signal. The low frequency part of $r(t)$ consists of a sum of five unit steps, at $t = 10$, 110, 210, 310, and 410, and the high frequency part is white Gaussian noise with standard deviation 0.5. In Fig. 2 $y_d(t)$ is depicted together with the closed-loop output using $u_g(t)$ (30)to control system (26). It is clear that the performance is better at high and low amplitudes where the linear approximations hold.

The goal is now to find better parameter values for the gain scheduling controller. The linear controllers (28) and (29) give rise to the initial parameter value vector

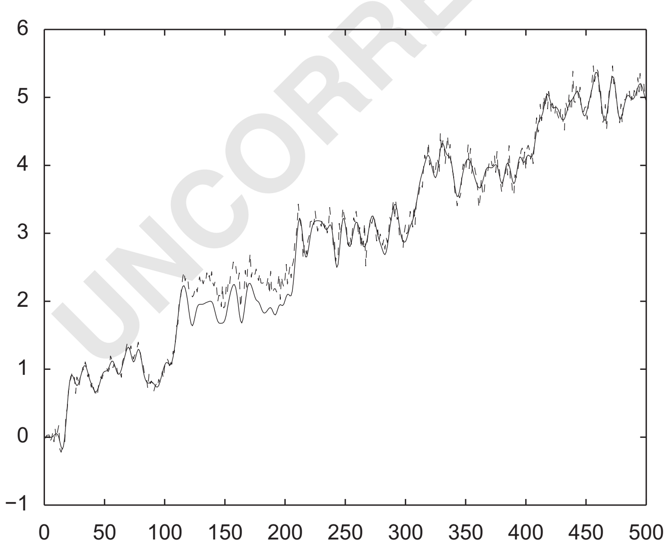

Fig. 2. Desired (solid) and true (dashed) plant output versus time when the system is controlled by the gain-scheduling controller in (30).

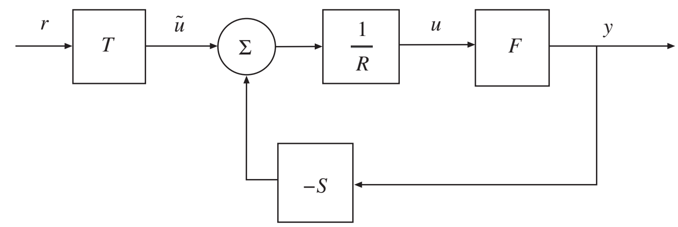

Fig. 3. The relation between $\tilde{u}(t)$ and $y(t)$ is described by the time-varying filter $(1/(1 + F(1/R)S))F1/R$. Hence, by using these signals as input and output in the identification gives us the needed filter directly.

Table 1
The value of the criterion of fit $J(\rho)$ for the initial gain-scheduling controller and after each of two iterations

Iteration #	$J(\hat{\rho})$
0	0.18
1	0.13
2	0.12

$$\rho_0 = [-0.6\;\; 0.36\;\; 0.3\;\; -2.1\;\; 2.06\;\; 0.3].$$

The control performance criterion is chosen

$$J(\rho) = \sum_t (y_d(t) - y(t))^2.$$

To compute the derivative of the plant output with respect to the controller parameters using expression (17), the controller derivative along the trajectory and the time-varying filter $(1/(1 + F(1/R)S))F(1/R)$ are needed. Using (30), (28), and (29) one obtains

$$g(\{y_i, u_i, r_i\})' = [(1-k)[y(t)\;\; y(t-1)\;\; r(t)]\;\; k[y(t)\;\; y(t-1)\;\; r(t)]].$$

Instead of estimating a time-varying model F of the plant and then computing the filter, the filter $(1/(1 + F(1/R)S))F1/R$ is estimated directly. This can be done by using the linearized closed-loop system of (15) for $\Delta\rho = 0$, giving

$$\Delta y(t) = \frac{1}{1 + F\frac{1}{R}S} F\frac{1}{R} T\Delta r(t) + v'(t) + O(\Delta^2_{\max}), \tag{31}$$

where $v'(t) = (1/(1 + F(1/R)S))v(t)$. Eq. (31) has the same structure as Eq. (13) with F replaced by $(1/(1 + F(1/R)S))F(1/R)$, $u(t)$ replaced by $\tilde{u}(t) = \mathrm{T}r(t) = (\partial u_g(t)/\partial r(t))r(t) = ((1-k)\rho_3 + k\rho_6)r(t)$, and $v(t)$ replaced by $v'(t)$. The filter $(1/1 + F(1/R)S)F(1/R)$ can therefore be estimated through the linear time-varying identification approach for F described in Section 5 by using the signals $y(t)$ and $\tilde{u}(t)$ instead of $y(t)$ and $u(t)$. This is illustrated in Fig. 3.

A time-varying ARX model of the form (23) is used with

$$\varphi(t) = [-y(t-1)\;\; -y(t-2)\;\; \tilde{u}(t-1)]^{\mathrm{T}}$$

and the model-parameter estimate is defined with criterion (25) with a constant forgetting factor $\lambda_t = 0.95$.

In Table 1 the root-mean-square error J is shown for the initial gain-scheduling controller and after each of two iterations of Algorithm 1.1. A criterion decrease of 30% was obtained by the tuning. Indeed, in this case, the algorithm converges to the vicinity of the optimum after two iterations. Due to the noise $v(t)$ in (26), further iterations will continue to give results in the vicinity of the optimum, but without strict convergence.

The parameters of the tuned controller are

$$\rho = [-0.85\;\; 0.44\;\; 0.42\;\; -2.1\;\; 2.0\;\; 0.35].$$

The control performance of the final controller is depicted in Fig. 4. Clearly, while achieving a lower value of $J(\rho)$, the final tuned

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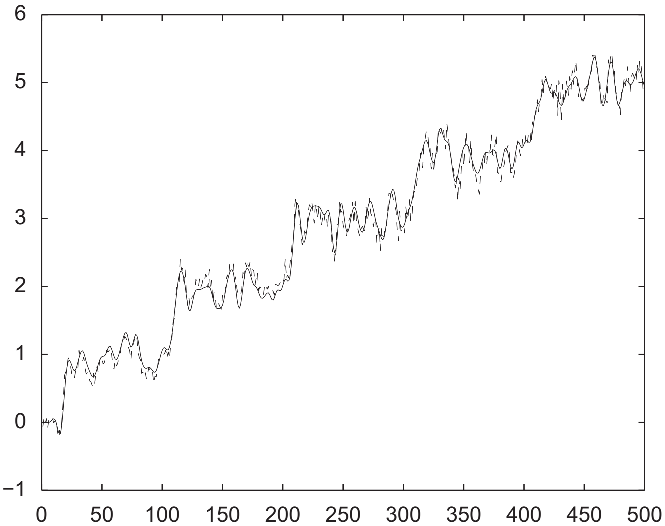

Fig. 4. Final tuned gain-scheduling controller: desired (solid) and true (dashed) plant output.

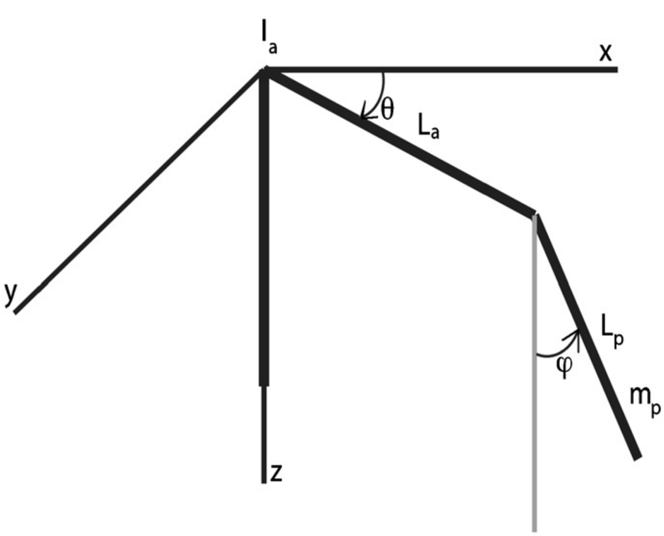

Fig. 5. A sketch of the Furuta pendulum.

controller improves the performance at mid-range and has a more balanced performance across the complete operating range, in comparison with the original gain-scheduling controller (30). The latter has almost perfect knowledge of the plant at low and high output values which explains its good performance at each end, see Fig. 2.

7.2. Tuning of a PID controller for the Furuta pendulum

The Furuta pendulum (Furuta, Yamakita, & Kobayashi, 1991; Furuta, Yamakita, Kobayashi, & Nishimura, 1991) has a rotating axle on which a perpendicular arm is fixed from which a pendulum is hinged, as illustrated in Figs. 5 and 6. Its equations of motion, derived in, e.g. Furuta, Yamakita, and Kobayashi (1991) and Bax (2007), are

$$I_a\ddot{\theta} + m_p l_a^2\ddot{\theta} + \tfrac{1}{4} m_p l_p^2 \ddot{\theta}\sin^2\varphi + \tfrac{1}{4} m_p l_p^2 \dot{\theta}\dot{\varphi}\sin 2\varphi - \tfrac{1}{2} m_p l_a l_p \ddot{\varphi}\cos\varphi + \tfrac{1}{2} m_p l_a l_p \dot{\varphi}^2 \sin\varphi + C_a\dot{\theta} = M, \qquad (32)$$

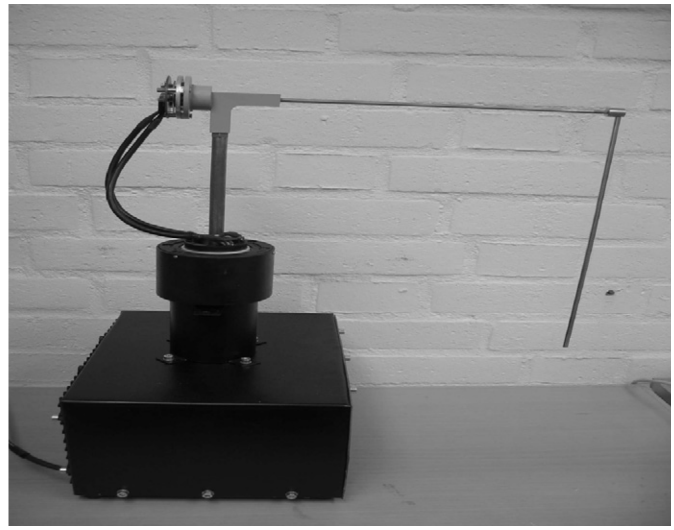

Fig. 6. The experimental Furuta pendulum in the laboratory.

$$I_p\ddot{\varphi} + \tfrac{1}{4} m_p l_p^2 \ddot{\varphi} - \tfrac{1}{2} m_p l_a l_p \ddot{\theta}\cos\varphi - \tfrac{1}{8} m_p l_p^2 \dot{\theta}^2 \sin 2\varphi + \tfrac{1}{2} m_p g l_p \sin\varphi + C_p\dot{\varphi} = 0, \qquad (33)$$

where the arm angle $\theta(t)$ (rad) and pendulum angle $\varphi(t)$ (rad), defined in Fig. 5, are functions of time t (s), $M(t)$ (Nm) is the torque delivered from the electrical DC-motor, I_a (kg m^2) is the moment of inertia of the axle and arm assembly around its center of mass , m_p (kg) is the mass of the pendulum with the center of mass at half length of the pendulum, l_a (m) is the arm length, l_p (m) is the pendulum length, C_a (Nm/(rad/s)) is the viscous friction coefficient in the axle joint, I_p (kg m^2) is the pendulum moment of inertia around its center of mass, g (m/s^2) is the acceleration due to gravitation, and C_p (Nm/(rad/s)) is the viscous friction coefficient of the pendulum joint. I_a, l_a, l_p, and m_p are suggested in Fig. 5. Nonlinear effects such as nonlinear friction are neglected.

The rotating axle is driven in an H-bridge configuration by the DC-motor which may be approximated as a current drive for low rotational velocities. Hence $M(t) \approx k_M i(t)$, where k_M (Nm/A) is the DC-motor torque constant, and $i(t)$ is the electrical motor current.

Assuming a constant arm rotational velocity $\dot{\theta}_0 > 0$, such that $2g/(l_p\dot{\theta}_0^2) \hat{=} d \leqslant 1$, there exist two equilibria with the corresponding constant driving torque, $M_0 = \dot{\theta}_0/C_a$, and constant pendulum angle φ_0 given by $\cos(\varphi_0) = d$. Hence, in one equilibrium, $0 \leqslant \varphi_0 < \pi/2$, and the pendulum is hanging "backwards" relative to the motion of the arm, see Fig. 5. It will be shown below that the linearized transfer function around this equilibrium is asymptotically stable, and minimum phase. In the second equilibrium, $-\pi/2 < \varphi_0 < 0$, and the pendulum is hanging "forwards" relative to the motion of the arm. The linearized transfer function around the second equilibrium is asymptotically stable, but non-minimum phase. Motions around these equilibria, referred to as the MP case, and NMP case, respectively, will be the subject of this example. Other possible equilibria are investigated in, e.g., Bax (2007).

The linearized transfer functions around the two equilibria discussed above are

$$\Delta\varphi = \frac{B(s)}{D(s)}\Delta\dot{\theta} \quad \text{with } \Delta\dot{\theta} = \frac{D(s)}{A(s)}\Delta M, \qquad (34)$$

where $\Delta\varphi = \varphi - \varphi_0$, $\Delta\dot{\theta} = \dot{\theta} - \dot{\theta}_0$, $\Delta M = M - M_0$, and

$$
\begin{aligned}
B(s) &= m_p l_a l_p d s/2 \pm m_p l_p^2 \dot{\theta}_0 d\sqrt{1-d^2}/2,\\
D(s) &= (I_p + m_p l_p^2/4)s^2 + C_p s + (m_p g l_p d/2 - m_p l_p^2 \dot{\theta}_0^2 (2d^2-1)/4),\\
A(s) &= ((I_a + m_p l_a^2 + m_p l_p^2(1-d^2)/4)(I_p + m_p l_p^2/4) - (m_p l_a l_p d/2)^2)s^3\\
&\quad + ((I_a + m_p l_a^2 + m_p l_p^2(1-d^2)/4)C_p + C_a(I_p + m_p l_p^2/4))s^2\\
&\quad + ((I_a + m_p l_a^2 + m_p l_p^2(1-d^2)/4)(m_p g l_p d/2\\
&\quad - m_p l_p^2 \dot{\theta}_0^2 (2d^2-1)/4) + C_a C_p\\
&\quad + (m_p l_p^2 \dot{\theta}_0 d\sqrt{1-d^2}/2)^2)s + C_a(m_p g l_p d/2\\
&\quad - m_p l_p^2 \dot{\theta}_0^2 (2d^2-1)/4). \qquad (35)
\end{aligned}
$$

In $B(s)$, the plus sign in front of the constant term belongs to the MP case, and the minus sign to the NMP case. The roots of $D(s)$ and $A(s)$ are all in the left half of the complex plane for a neighborhood of the nominal parameter values, $I_a = 0.0016$, $m_p = 0.019$, $l_a = 0.33$, $l_p = 0.353$, $C_a = 0.01$, $I_p = 0.000789$, and $C_p = 0.001$, and hence the transfer functions from ΔM to $\Delta\varphi$, and from $\Delta\dot{\theta}$ to $\Delta\varphi$ are asymptotically stable. The plant model (32), (33) is nonlinear which makes the transfer functions (34), (35) depend also on the operating point expressed by $\dot{\theta}_0$. Hence it could be of interest to tune a feedback controller for various operating conditions.

The to be controlled measured output signal is the pendulum angle $y(t) = \varphi(t)$, and the control signal is the motor current $u(t) = i(t)$. The reference signal is $r(t)$ (rad), and the error is defined as $e(t) = r(t) - y(t)$. The criterion to be minimized by controller tuning was chosen as

$$J = \sum_t e^2(t)/2, \qquad (36)$$

where it is noted that $e(t)$ and J depend on the controller and its parameters, defined below. J is computed over a time interval

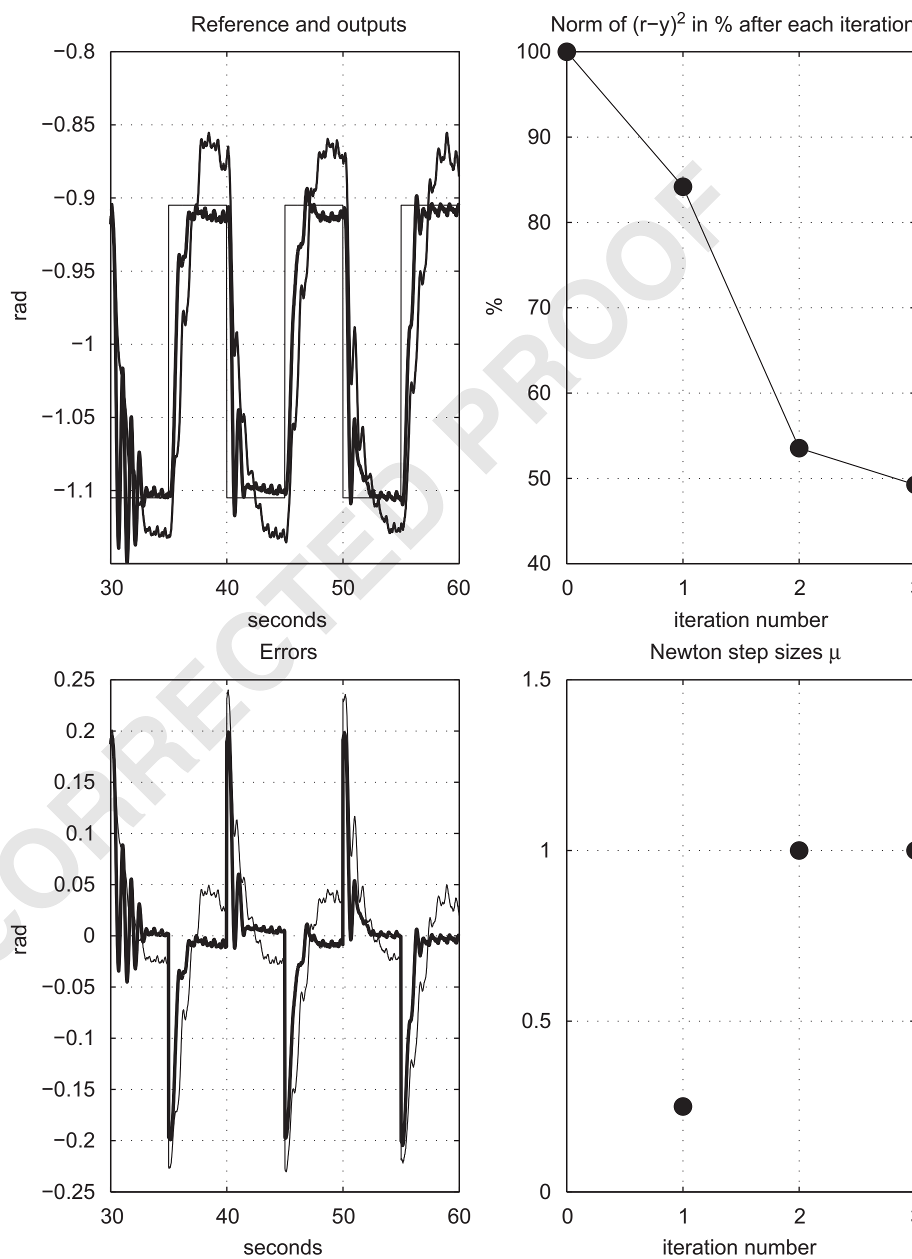

Fig. 7. Upper left: the square wave is the reference angle; the thin line is the arm angle with initial controller tuning; the bold face line is the arm angle with final controller tuning. Lower left: loop error $e(t)$ for initial tuning (thin) and final tuning (bold). Upper right: the criterion value (36). Lower right: the step size μ in (6).

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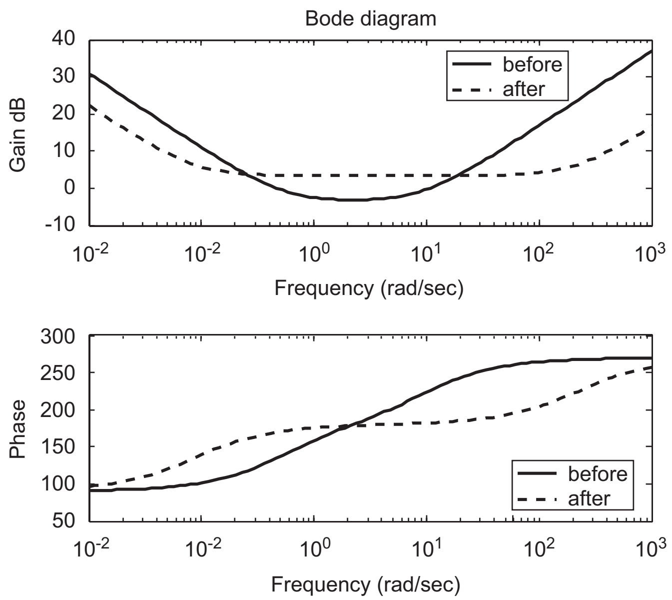

Fig. 8. Bode diagram of the analog counterparts, $U(s)=k_p(1+1/\tau_i s+\tau_d s)E(s)$ of the initially tuned, and final PID-controllers. Note that the Nyquist frequency is 10π (rad/s).

when the pendulum exhibits "steady state" behavior, after the passage of the start-up transient. In contrast to the general criterion in (4) the reference signal is not filtered, and the control effort is not penalized.

The feedback controller was chosen to be a standard PID, $U(s)=k_p(1+1/\tau_i s+\tau_d s)E(s)$, where $U(s)$ and $E(s)$ are the Laplace transforms of $u(t)$ and $e(t)$, respectively, transformed to discrete time form by replacing differentiation with the backward difference:

$$u(t)=u(t-t_s)+e(t)k_p(1+\tau_d/t_s) \\ +e(t-t_s)k_p(t_s/\tau_i-1-2\tau_d/t_s)+e(t-2t_s)k_p\tau_d/t_s, \quad (37)$$

where t is the sample time (s), and $t_s=0.1$ s is the sampling interval. Hence, the parameter vector to be tuned is

$$\rho=[k_p,\tau_i,\tau_d]^{\mathrm{T}}. \quad (38)$$

In the experimental NMP case, the standard Ziegler–Nichols tuning rule was attempted for the initial controller tuning; however, the closed loop became unstable, and the integral action had to be weakened. The stabilizing initial controller parameter vector was determined as $\rho_0=[-0.7,2,0.1]^{\mathrm{T}}$. See Figs. 7 and 8.

An input–output data sequence comprising 1000 samples, i.e., 100 s, was recorded in closed loop, with no extra noise added, since the laboratory equipment "produced" sufficient inherent noise. Following Eq. (31) and Fig. 3, a time-varying linear model for the closed loop $(TF/R)/(1+SF/R)$ from $r(t)$ to $y(t)$, is identified off-line, assuming model (22) with $n_a=3$, $n_b=2$, and $n_k=1$. For each time instant t, the coefficients of A and B in (22) were obtained by minimizing the weighted least squares criterion

$$V(a_1(t),\ldots,b_2(t),t)=\frac{1}{2}\sum_{i=1}^{n}\lambda^{|t-i|}(y(i)-\hat{y}(i))^2 \quad (39)$$

with $n=1000$, the weighting factor $\lambda=0.95$, and $\hat{y}(i)$ denoting the model output. In order to perform the computation in equation (17), the known signal g' is filtered through the identified model representing $(TF/R)/(1+SF/R)$, in series with the known filter $1/T$, see (11), and $y'(t)$ is obtained.

Now an update of ρ takes place according to (6) where the step length μ is initially chosen to 1, and then decreased by half each time, until criterion (36) decreases. The procedure is iterated with the newly tuned controller as the initial one, until a satisfactory reduction of the criterion is achieved. The final controller parameter vector was found to be $\rho_3=[-1.4799,11.6068,0.0045]^{\mathrm{T}}$. The results of this challenging experimental NMP case are illustrated with Figs. 7 and 8. In this experiment, a reduction of the criterion by 50% is achieved, in three iteration steps, by tuning a standard, but initially quite badly tuned, PID controller.

More simulations and experimental results in both the MP and NMP cases are found in Bax (2007). The rate of convergence to a local minimum of the criterion seems to depend on the initial controller tuning, the noise level, and how faithfully the true plant is modeled. In Bax (2007) a web site is indicated from where movies of the controlled pendulum can be downloaded.

8. Conclusions

A new algorithm for tuning of controllers of nonlinear systems is presented. The controller parameters are iteratively updated so that a user-defined control criterion is minimized. In each iteration an experiment on the plant is performed with a specific reference signal. The data from the experiment are used to identify a linearized time-varying model along the reference trajectory, which is then used to calculate the gradient of the control criterion with respect to the control parameters.

Advantages with this approach are

- A global nonlinear model of the plant is not needed.
- The plant is modeled only where it is necessary—along the trajectory of the reference signal.

A small simulation example with a tuning of conventionally obtained gain scheduler, and the tuning of a PID controller for a laboratory set-up of the Furuta pendulum are given as illustrations of the proposed method.

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